Belief Propagation for Spatial Network Embeddings

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Outline

1. Graphical Models
   - Markov Random Fields
   - Inference
2. Self-Localization
   - Problem Description
   - Model Formulation
   - Experimental Results
3. Latent Space Embeddings of Social Networks
   - Problem Description
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   - Preliminary Results

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What Are Graphical Models?

- Concise representations of probabilistic models
- Several types:
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  - Markov random fields (undirected graphs)
  - Factor graphs (bipartite graphs)
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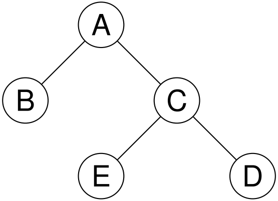

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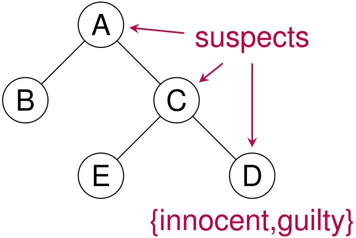

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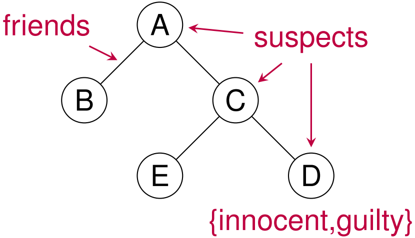

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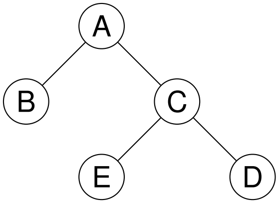
Representing Conditional Independencies

Interpreting a Markov Random Field

If all paths from X to Y pass through Z, then we can say
X and Y are conditionally independent given Z.

Graphically, with a
Markov Random Field (MRF):



Textually, through enumeration:

- $A \perp D, E \mid C$
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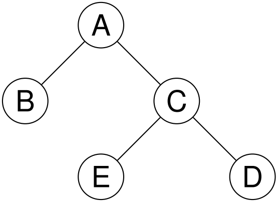
- $\ldots$

Factorization

- Conditional independence lets us factor a distribution:



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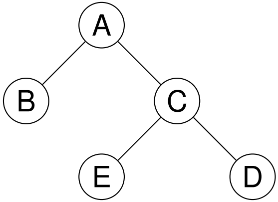

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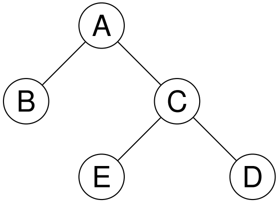

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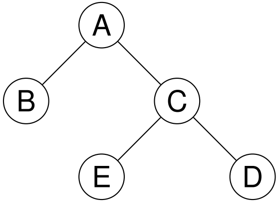

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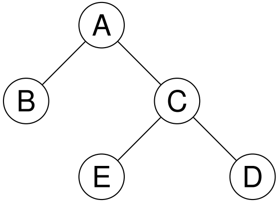

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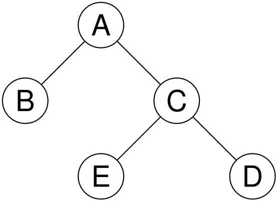

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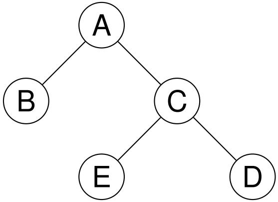

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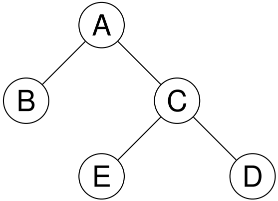

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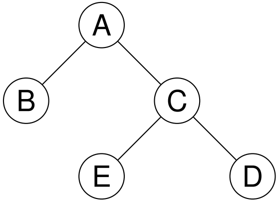

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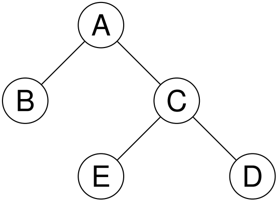

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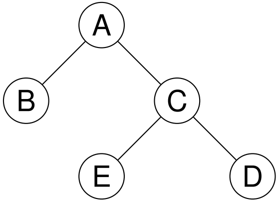

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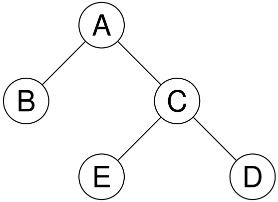

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Largest factor involves 2 variables!

Hammersley-Clifford Theorem

General factorization property of all MRFs:

Hammersley-Clifford Theorem

Every MRF factors as the product of potential functions defined over cliques of the graph.

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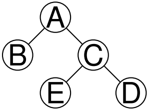

$$p(\cdot) \propto f_A(A) f_B(B) f_C(C) f_D(D) f_E(E) f_{AB}(A, B) f_{AC}(A, C) f_{CD}(C, D) f_{CE}(C, E)$$

Specifying a Markov Random Field Model

Define the potential functions, e.g.:
Let our domain be 0=innocent, 1=guilty.

$$f_B(B) = \begin{cases} .4 & B = 0 \\ .6 & B = 1 \end{cases}$$

$$f_{AB}(A, B) = \begin{cases} 2 & A = B \\ 1 & A \neq B \end{cases}$$

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Suspects A and B are friends.

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Query p(A):

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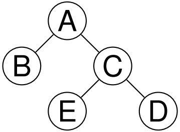
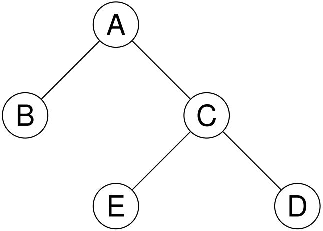


Use graph structure to compute p(A) in $O(dn^2)$.

Belief Propagation (Sum-Product Algorithm)

- View marginalization as a "message-passing" algorithm
  - Variables are computational nodes.
  - Intermediate results are "messages" between nodes.



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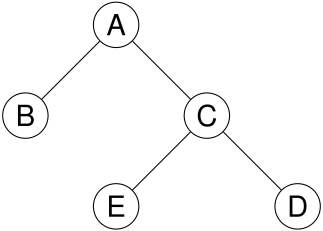

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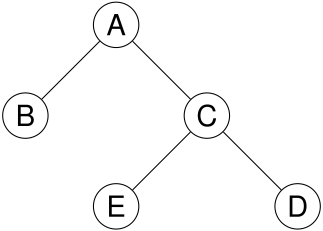

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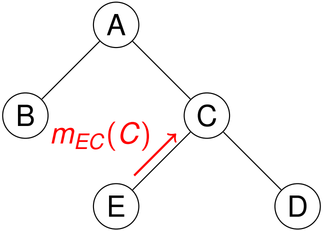

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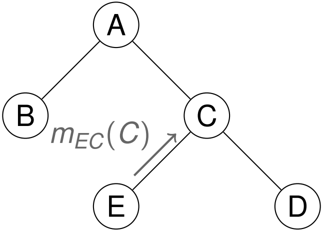

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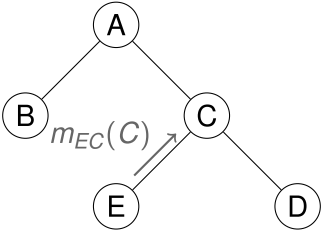

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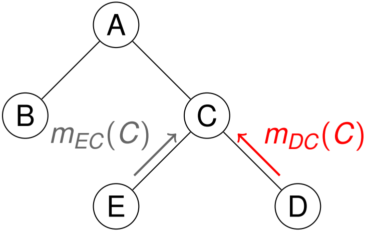

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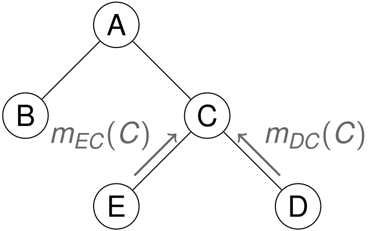

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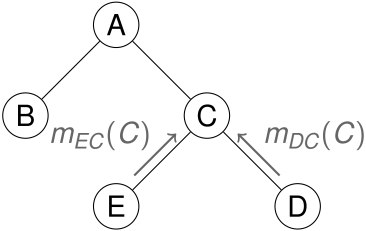

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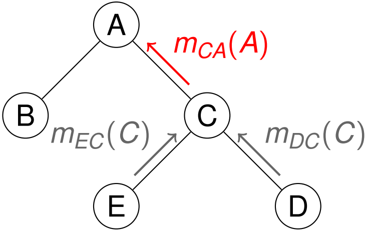

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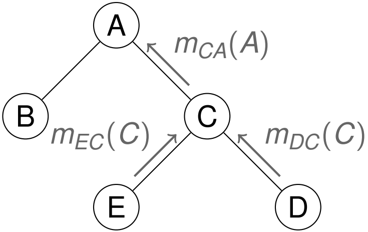

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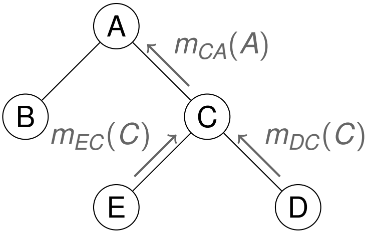

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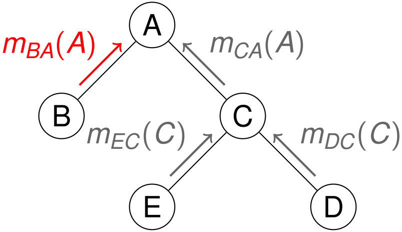

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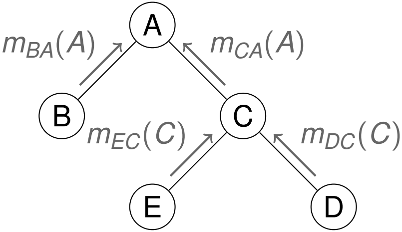
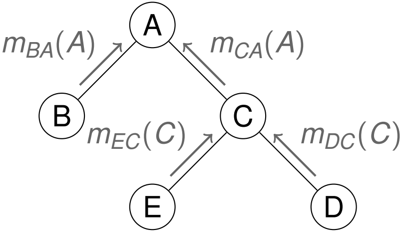


$f(A)m_{CA}(A)m_{BA}(A)$

Belief Propagation (Sum-Product Algorithm)

- View marginalization as a "message-passing" algorithm
  - Variables are computational nodes.
  - Intermediate results are "messages" between nodes.



$\propto p(A)$

Belief Propagation (Sum-Product Algorithm)

- Message update equation for pairwise MRFs:

$$m_{st}(x_t) = \sum_{x_s} \left[f(x_s) f(x_s, x_t) \prod_{x_u \in \mathcal{N}(x_s) \setminus x_t} m_{us}(x_s) \right]$$

  - Exact for tree-structured graphs.

Belief Propagation (Sum-Product Algorithm)

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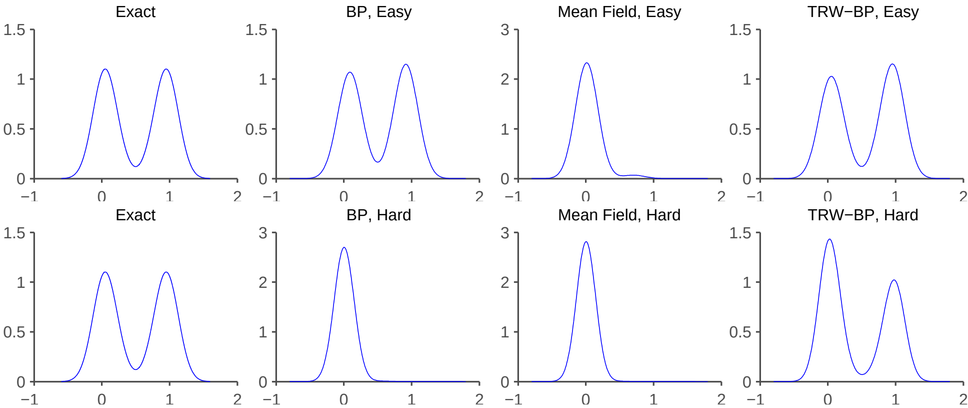
  - Exact for tree-structured graphs.
- What about on graphs with loops?
  - Use the same equation! (“Loopy” BP)
  - No longer exact
  - May not converge
  - Often does quite well

Related Algorithms



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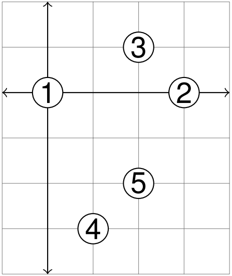
Localization Scenario

- Nodes distributed throughout a planar region.
  - (people, mobile sensors, ...)



Local measurements:

node	$\{$(*neighbor, distance*)$\}$
1	{}
2	{}
3	{}
4	{}
5	{}

- Nodes that are “close enough” can estimate distance between them.

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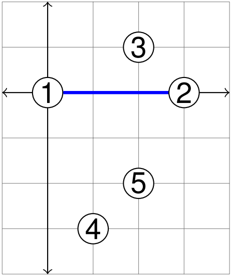

Local measurements:

node	{(*neighbor*, *distance*)}
1	{(2,3)}
2	{(1,3)}
3	{}
4	{}
5	{}

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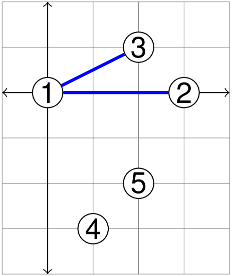

Local measurements:

node	{*(neighbor, distance)*}
1	{(2,3)(3,2)}
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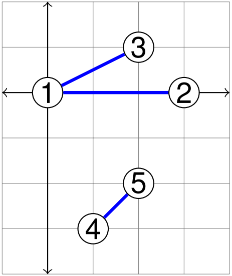

Local measurements:

node	{*(neighbor, distance)*}
1	{(2,3)(3,2)}
2	{(1,3)}
3	{(1,2)}
4	{(5,1)}
5	{(4,1)}

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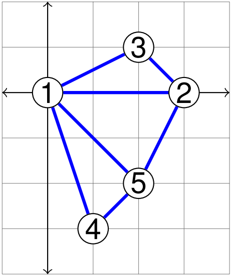

Local measurements:

node	{*(neighbor, distance)*}
1	{(2,3)(3,2)(4,4) (5,3)}
2	{(1,3)(3,1) (5,3)}
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4	{(5,1)(1,4)}
5	{(4,1)(1,3) (2,3)}

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Localization Scenario

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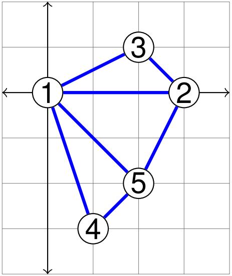

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- Nodes that are “close enough” can estimate distance between them.
- Task: recover node locations.

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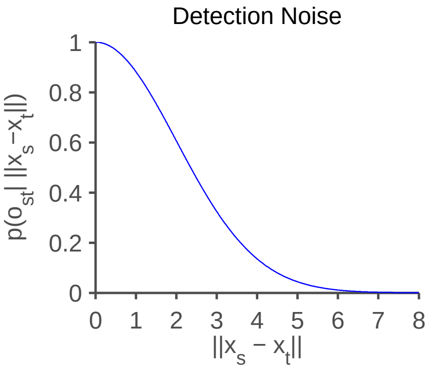
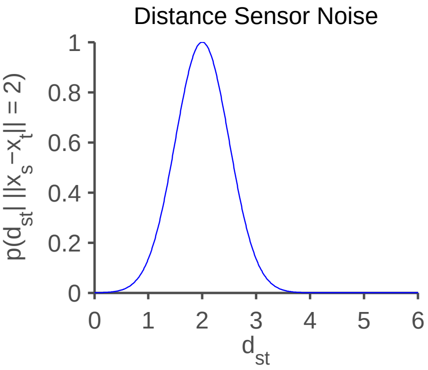
Local Detection Model

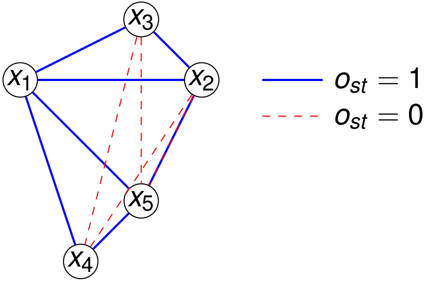
- Variables:
  - x_s, location in $\mathbb{R}^2$ of node s
  - o_{st}, indicates whether nodes s and t detect each other
  - d_{st}, noisy observation of $||x_s - x_t||$





Joint Model

$$p(\boldsymbol{x}, \boldsymbol{o}, \boldsymbol{d}) = \prod_{(s,t)} p(o_{st} \mid x_s, x_t) \prod_{(s,t):o_{st}=1} p(d_{st} \mid x_s, x_t) \prod_{s} p(x_s)$$



$$f_{st}(x_s, x_t) = \begin{cases} p(o_{st} = 1 | x_s, x_t) p(d_{st} | x_s, x_t) & \text{if } o_{st} = 1 \\ 1 - p(o_{st} = 1 | x_s, x_t) & \text{if } o_{st} = 0 \end{cases}$$

Handling Continuous Variables

- Variable domains are continuous! (locations in $\mathbb{R}^2$)

  $\Longrightarrow$ replace sums with integrals?

$$m_{st}(x_t) = \int_{x_s} \left[f(x_s) f(x_s, x_t) \prod_{x_u \in \mathcal{N}(x_s) \setminus x_t} m_{us}(x_s) \right]$$

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- Theory holds, but now we must compute the integrals.

Particle Belief Propagation (PBP)

- Draw weighted particles from each variable's domain.
- Run (importance-corrected) discrete BP over these particles.

$$m_{st}(x_t) = \int_{x_s} \left[f(x_s, x_t) f(x_s) \prod_{x_u \in \mathcal{N}(x_s) \setminus x_t} m_{us}(x_s) \right]$$

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$$
\begin{aligned}
m_{st}(x_t) &= \int_{x_s} \left[f(x_s, x_t) f(x_s) \prod_{x_u \in \mathcal{N}(x_s) \setminus x_t} m_{us}(x_s) \right] \\
&= \mathop{\mathbb{E}}_{x_s \sim W(x_s)} \left[f(x_s, x_t) \frac{f(x_s)}{W(x_s)} \prod_{x_u \in \mathcal{N}(x_s) \setminus x_t} m_{us}(x_s) \right]
\end{aligned}
$$

Particle Belief Propagation (PBP)

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$$m_{st}(x_t) = \int_{x_s} \left[f(x_s, x_t) f(x_s) \prod_{x_u \in \mathcal{N}(x_s) \setminus x_t} m_{us}(x_s) \right]$$

$$= \mathop{\mathbb{E}}_{x_s \sim W(x_s)} \left[f(x_s, x_t) \frac{f(x_s)}{W(x_s)} \prod_{x_u \in \mathcal{N}(x_s) \setminus x_t} m_{us}(x_s) \right]$$

$$\hat{m}_{st}^{(i)} \approx \frac{1}{n} \sum_{k=1}^{n} \left[f\left(x_s^{(k)}, x_t^{(i)}\right) \frac{f\left(x_s^{(k)}\right)}{W\left(x_s^{(k)}\right)} \prod_{x_u \in \mathcal{N}(x_s) \setminus x_t} m_{us}\left(x_s^{(k)}\right) \right]$$

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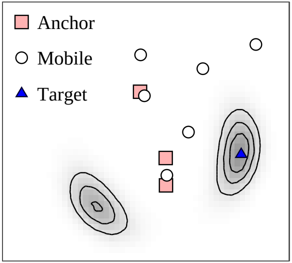
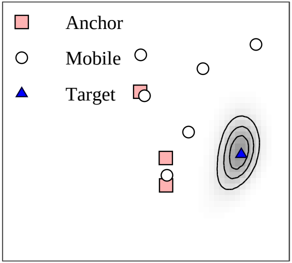
Results

Synthetic example:



“Exact”



Particle BP

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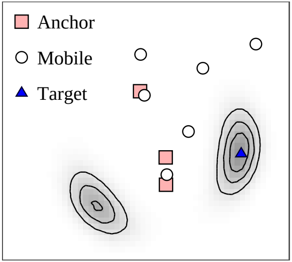

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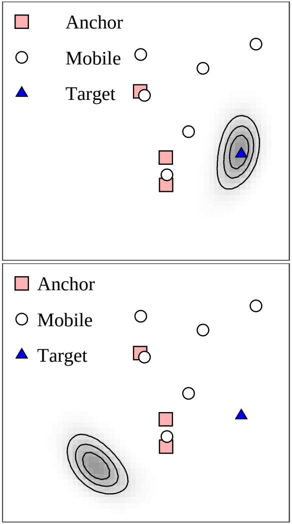

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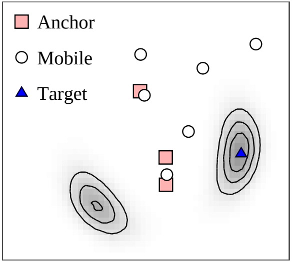

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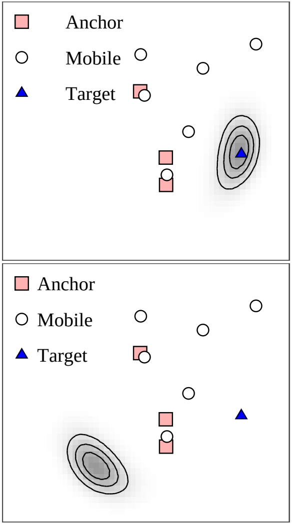
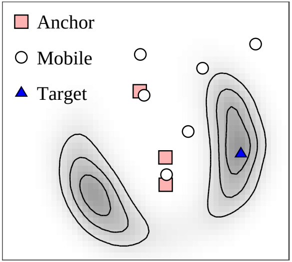


Particle BP



TRW Particle BP

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Main Idea

- Intuition
  - Actors live in a latent, $d-$dimensional "social space".
  - Proximity in social space increases the likelihood of a link.

Hoff, Raftery, Handcock. Latent space approaches to social network analysis. JASA 2002.

Connection to Localization

- Localization
  - Geographic space
  - Detection $\Rightarrow$ physical proximity
  - Location is the end goal
- Latent space embedding
  - Social space
  - Network link $\Rightarrow$ proximity in latent space
  - Latent location indirectly useful

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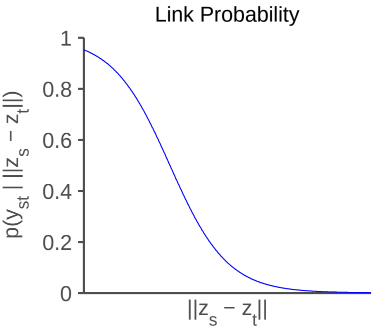
Local Model

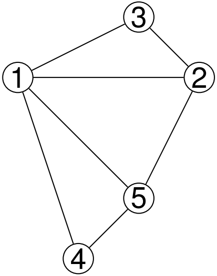
- Variables:
  - z_s, location in latent space of node s
  - y_{st}, social network link indicator

$$p(y_{st} \mid z_s, z_t) = \sigma(\alpha - ||z_s - z_t||)$$

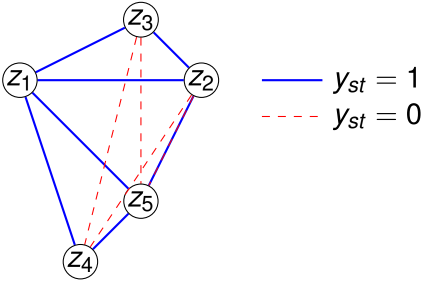


Joint Model

Social Network:



MRF model:



$$f_{st}(z_s, z_t) = p(y_{st} | z_s, z_t)$$

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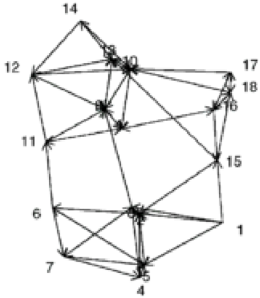
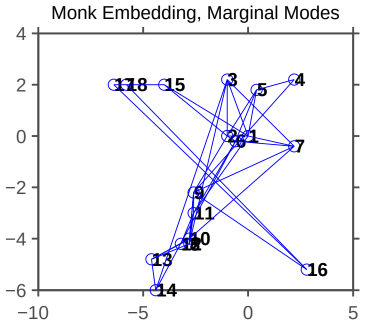
Test Data Set

- Sampson's monk data
  - 18 monks living in a monestary
  - Links indicate a "liking" relation
  - Well studied data set

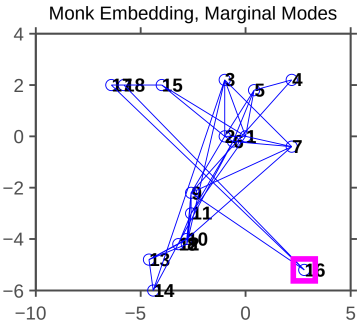


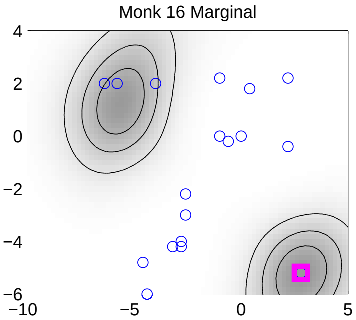
MLE (Hoff, Raftery, Handcock '02)

PBP Embedding of Monk Data



PBP Embedding of Monk Data





Conclusions

- BP is a generic inference method for computing marginals.
- PBP can estimate marginals in the self-localization problem.
- Could BP be useful for latent space network modeling?

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Thank you!