Statistical Models for Network Data: What and Why

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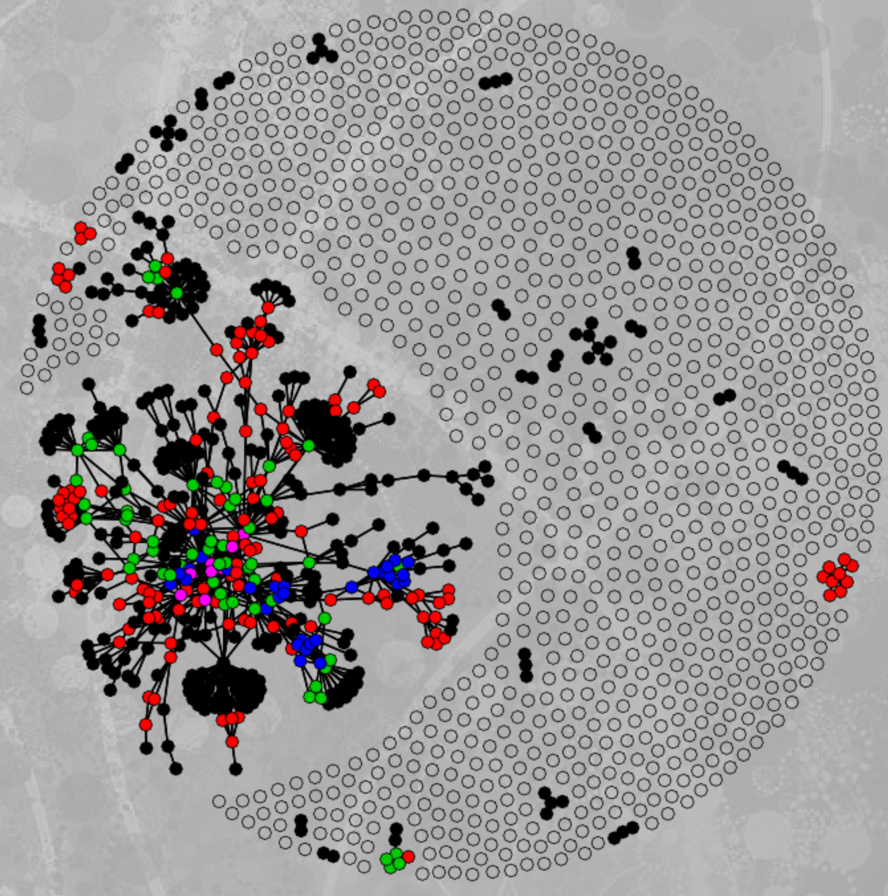
Introduction: The "What"

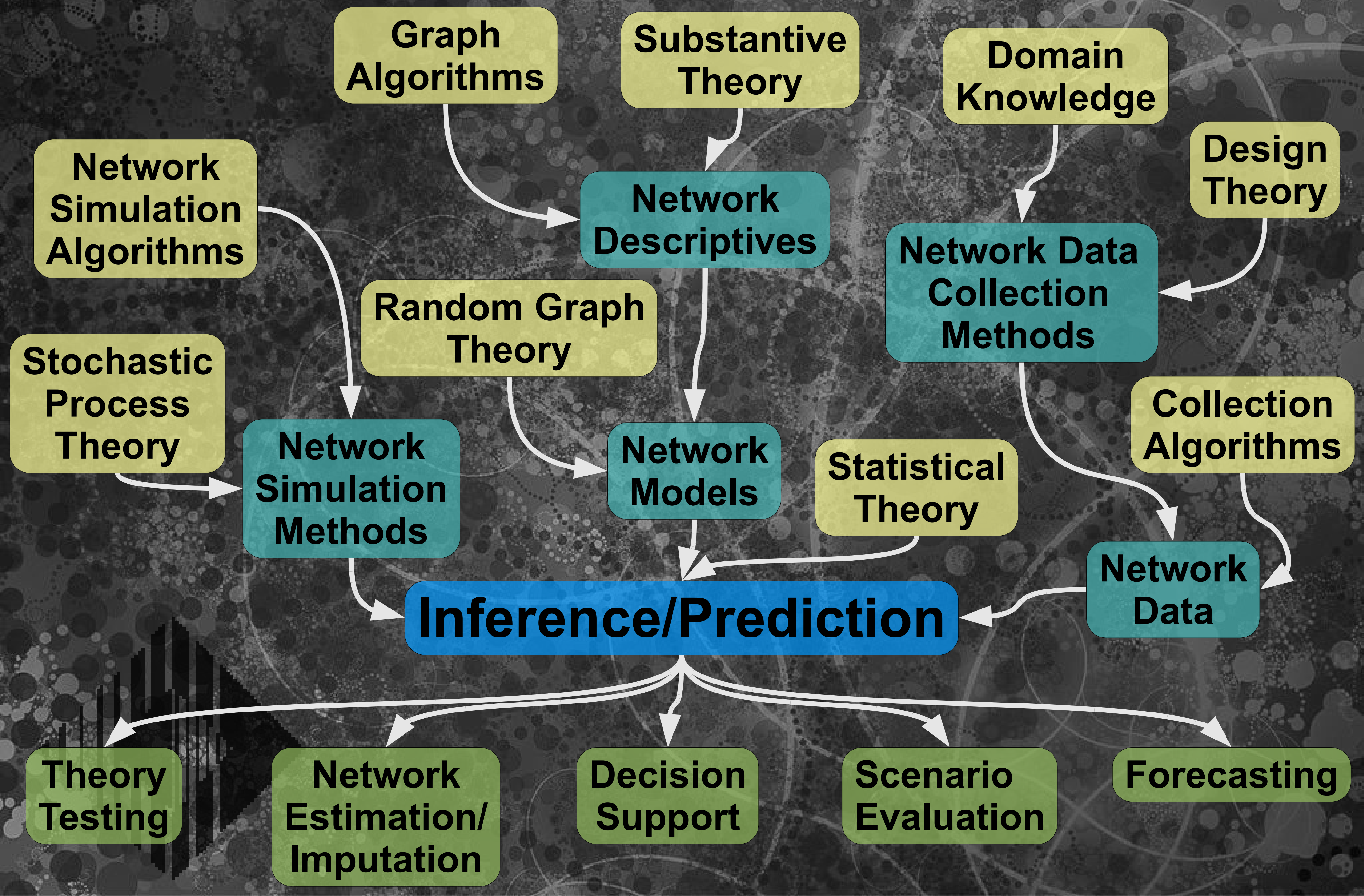
- The key questions regarding sociotechnical systems are relational
  - Connectivity, robustness, centrality, diffusion, etc.
- How do we make sense of this information?
- The statistical approach:
  - Assume that what we see reflects processes with many potential outcomes
  - Posit models that reflect our uncertainty about unknowns
  - Reason from observations and prior knowledge to unknown quantities in a principled manner



Key Challenges for this Approach

- Parameterizing models in a sensible and computable way
  - Models must reflect phenomenological understanding, but must also scale to real data
- Accounting for data collection
  - Need sampling methods, ways of handling missing/error-prone data
- Making inference both principled *and* practical
  - Want accurate estimates, but can't wait forever for results
- Dealing with rich, dynamic data
  - Real-world problems involve systems with complex covariates (text, geography, etc.) that change over time
- ***In sum: statistically principled methods that respect the realities of data and computational constraints***

Mapping the Project Terrain



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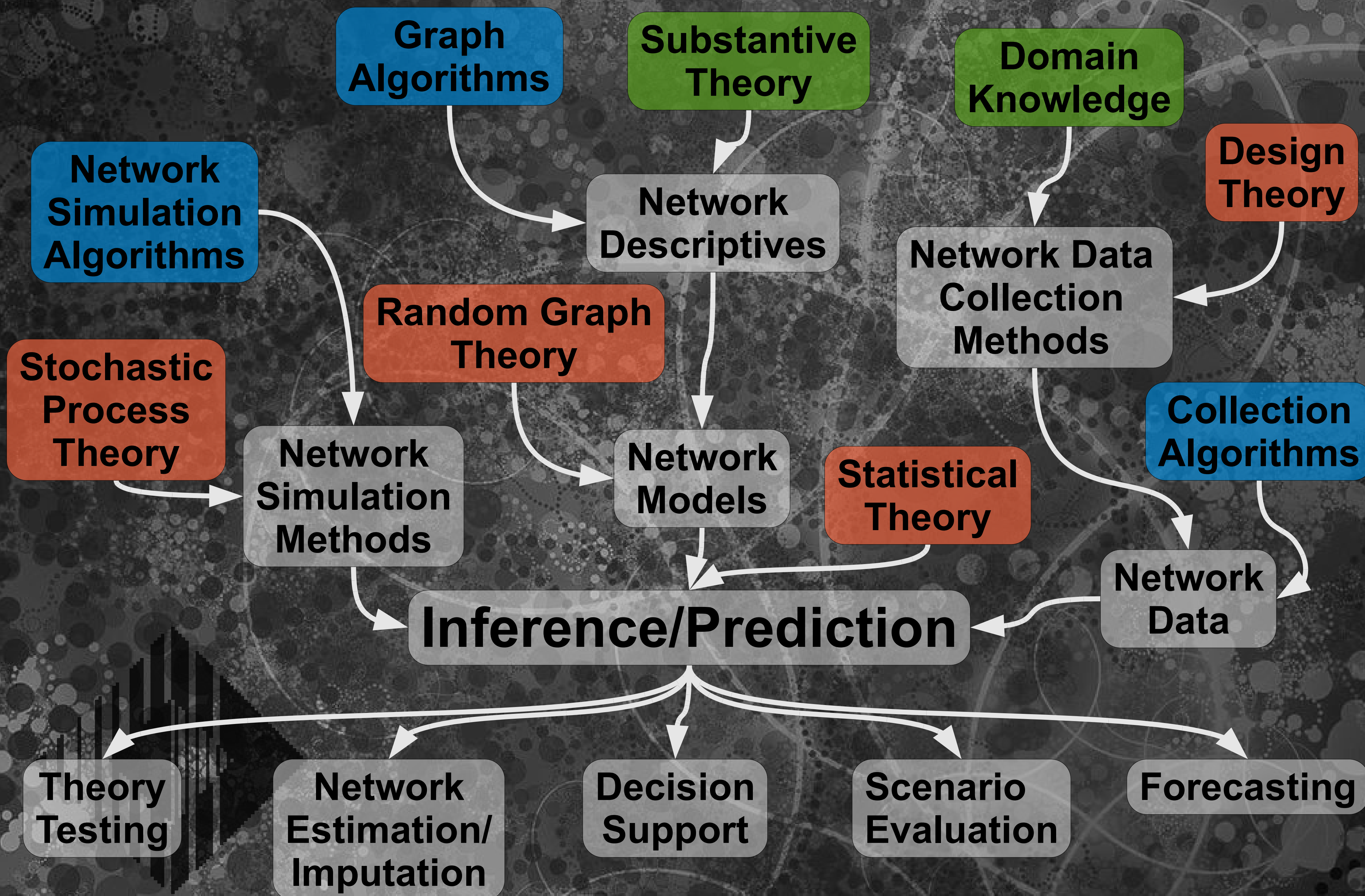

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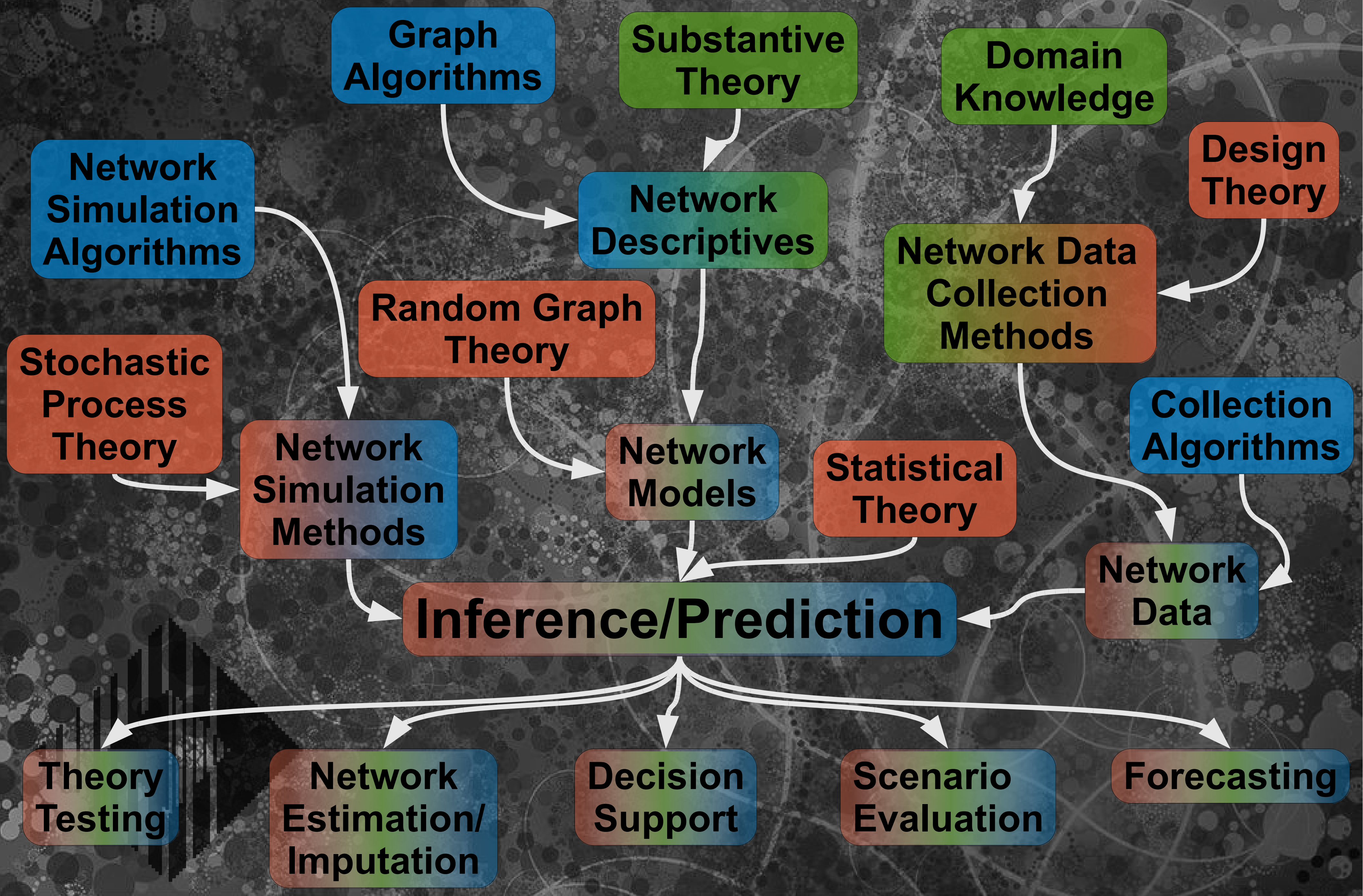
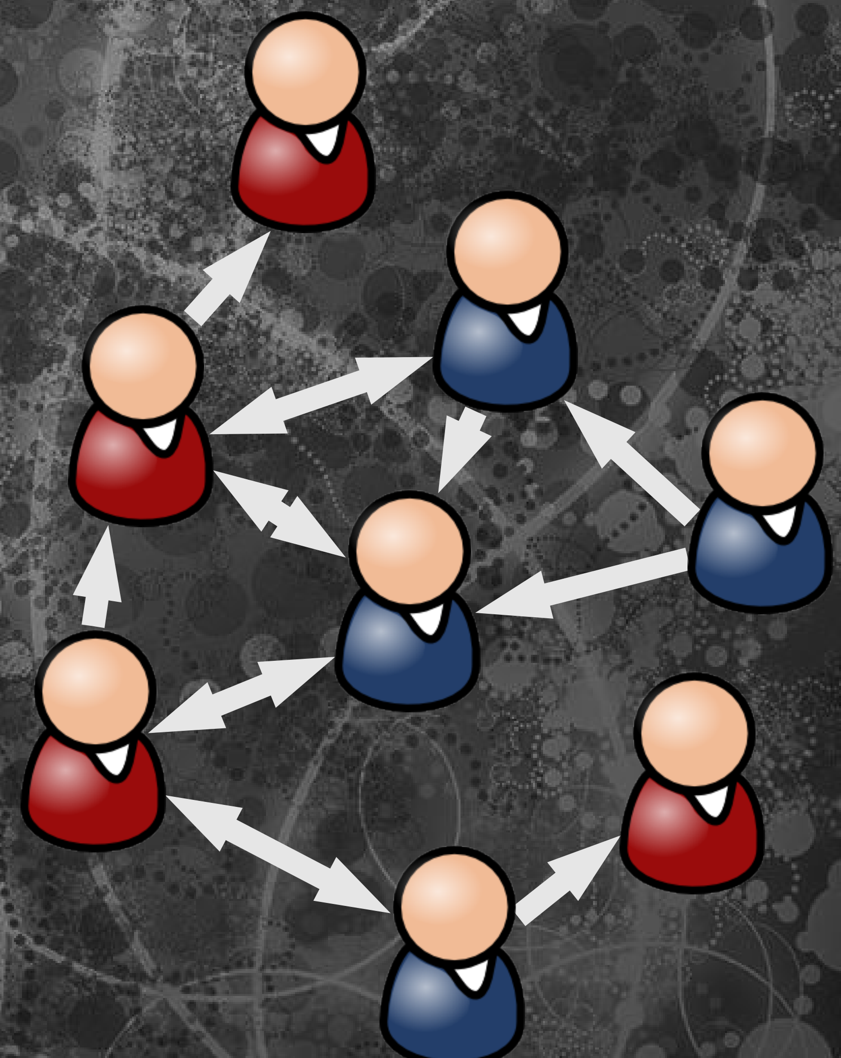


Why Statistical Models for Social (and Other) Networks?

- Social systems are complex
  - Many parts that affect each other
  - Substantial heterogeneity
- Many mechanisms involved
- We're not good at measuring them
  - Usually only see small chunks (and see above)
  - Error-prone observations
- Upshot: the network we see may result from many mechanisms, plus noise and unobserved factors
  - To draw conclusions about what is going on, must account for uncertainty
  - Predictions, conclusions should reflect this
  - Such goals require a statistical approach

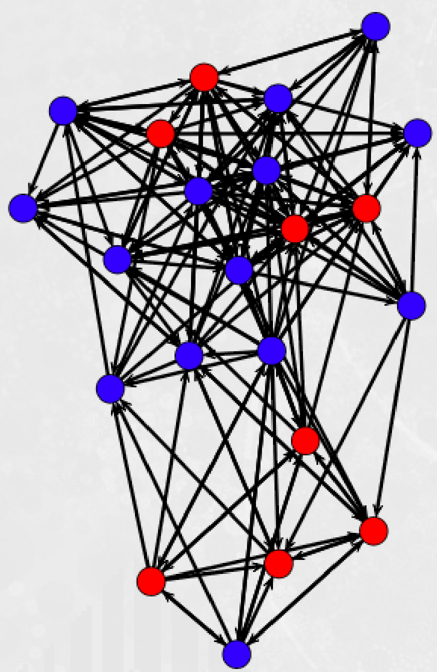
Motivating Example: The Reds and The Blues

- Consider a hypothetical community w/two groups - the "Reds" and the "Blues"
- Assume we are concerned with cooperation and trust in the community during a period of upheaval
- Our information is limited, but presume that we can observe networks of trust/friendship within representative subgroups....



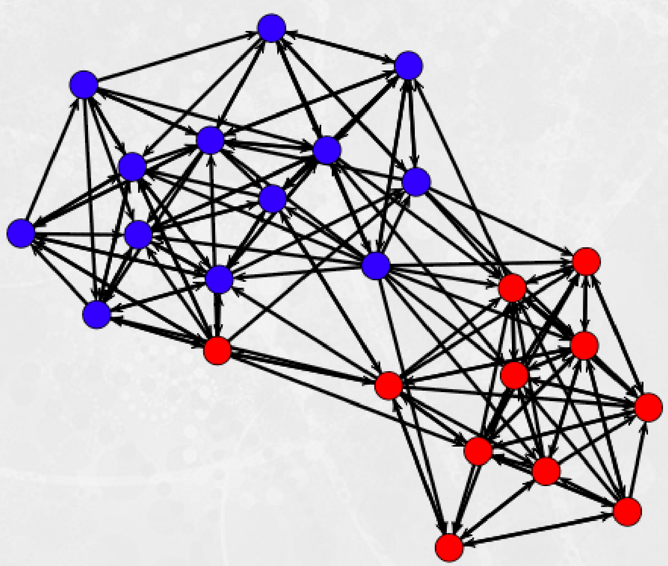
A Polarization Puzzle

Time 1



N=22

Time 2



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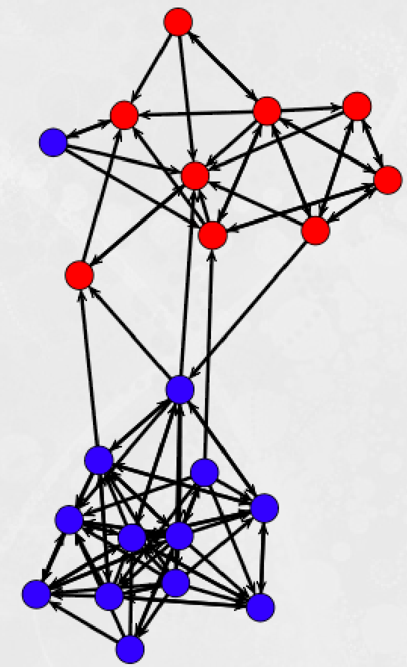
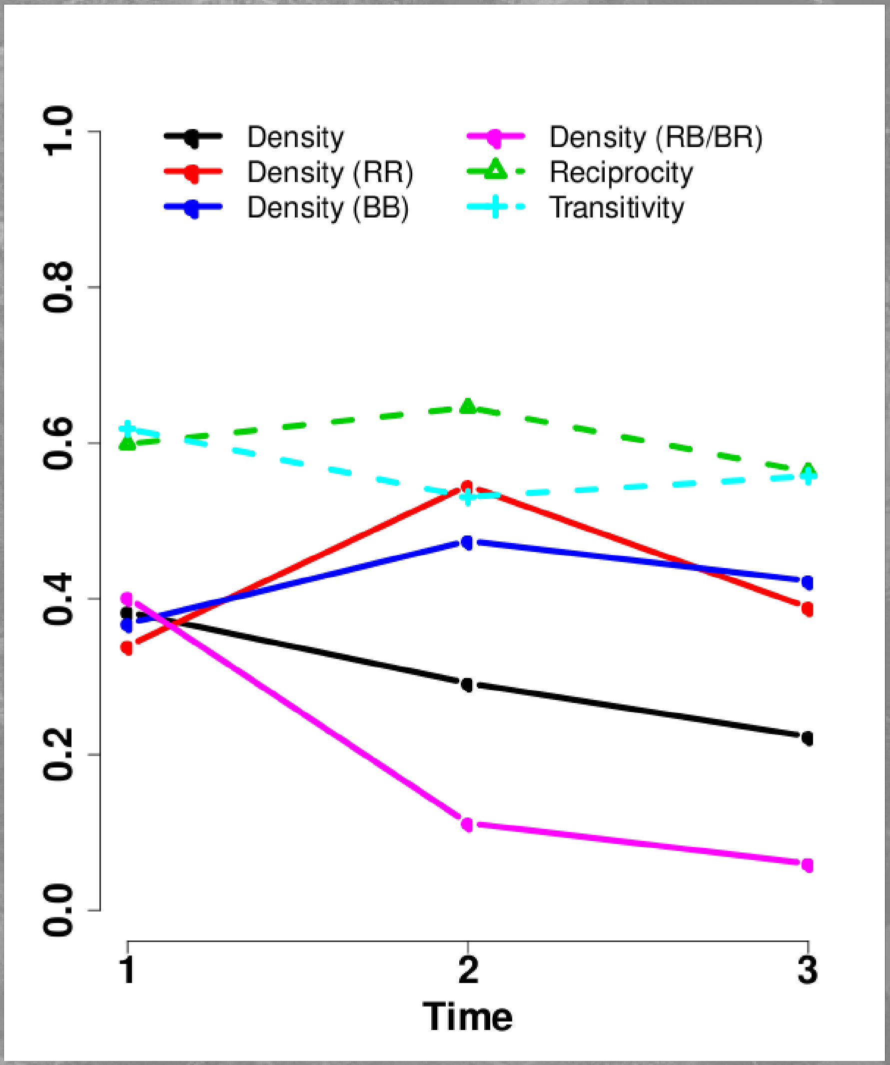
Time 3



N=22

First Step: Raw Descriptives

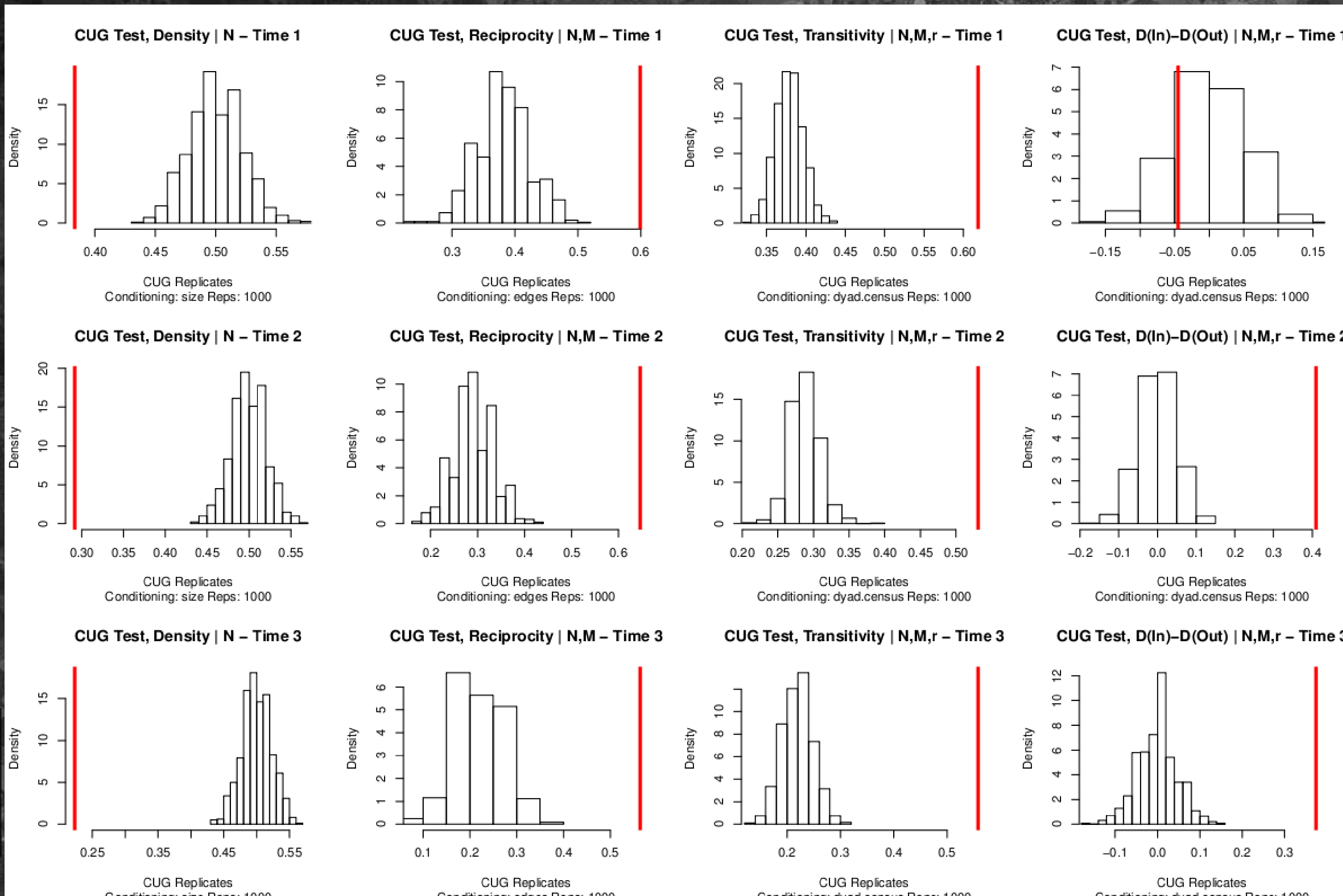
- Without a statistical approach, one is limited to description
- Here, some typical examples:
  - Density seems to fall slightly, although this masks an in/out-group difference
    - Red/Blue groups look similar
  - Moderately reciprocal, transitive networks, w/little change
- Gives a more precise accounting of events, but not very insightful
  - Are these changes even atypical of chance events?



Next Step: Baseline Models

- Slight refinement: compare network properties to simple "baseline" models
  - E.g., uniform random graphs, conditional on a few properties
- Most elementary statistical approach
  - Assesses whether combinatorics + elementary properties are sufficient to account for observations
- Allows us to ask simple, marginal questions
  - Is density atypical of population of all graphs given N?
  - Is reciprocity atypical of graphs given N,M?
  - Are transitivity, difference in in-group/out-group densities atypical of graphs given N,M,r?
- Compare to classical null hypothesis testing

Baseline Comparisons



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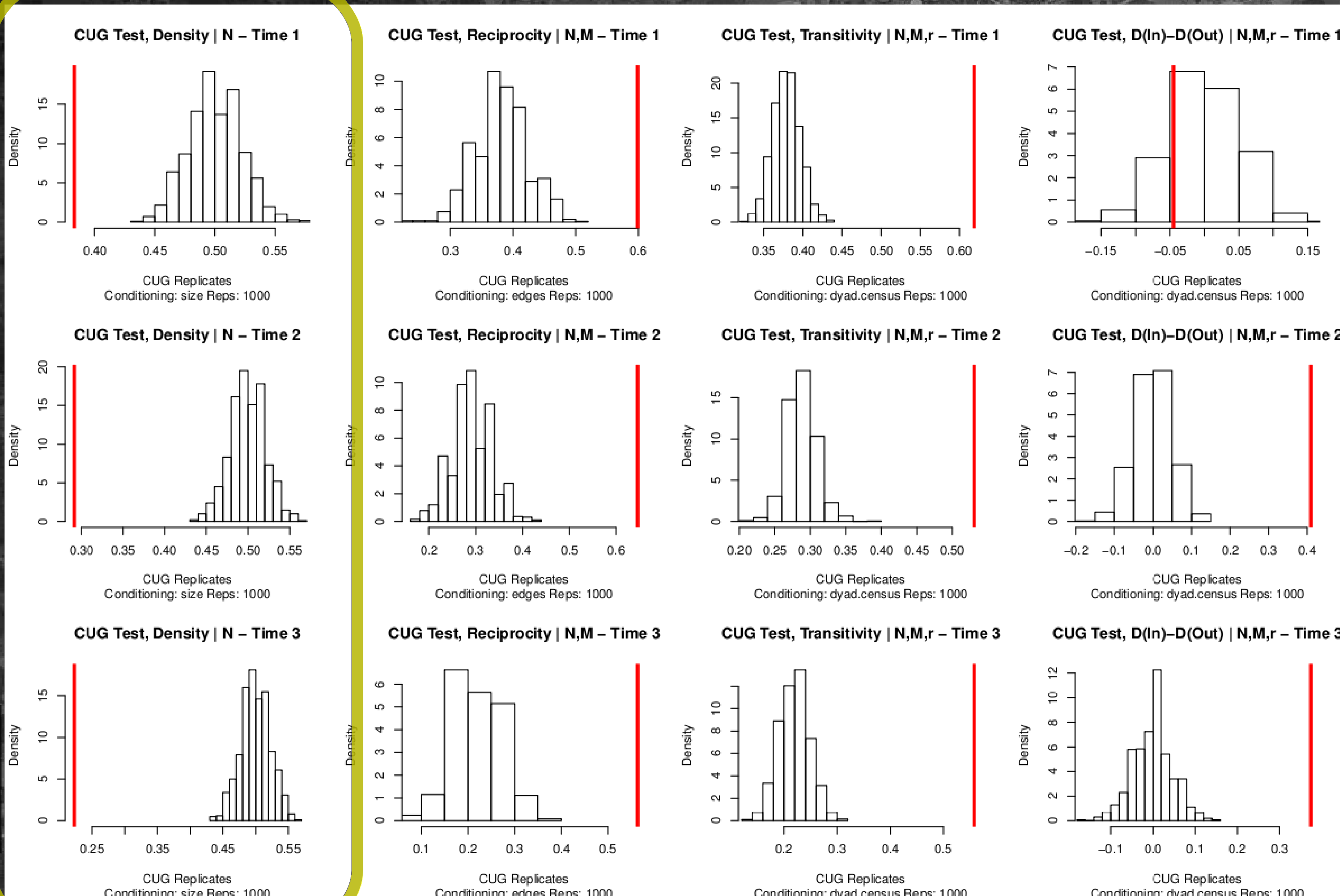

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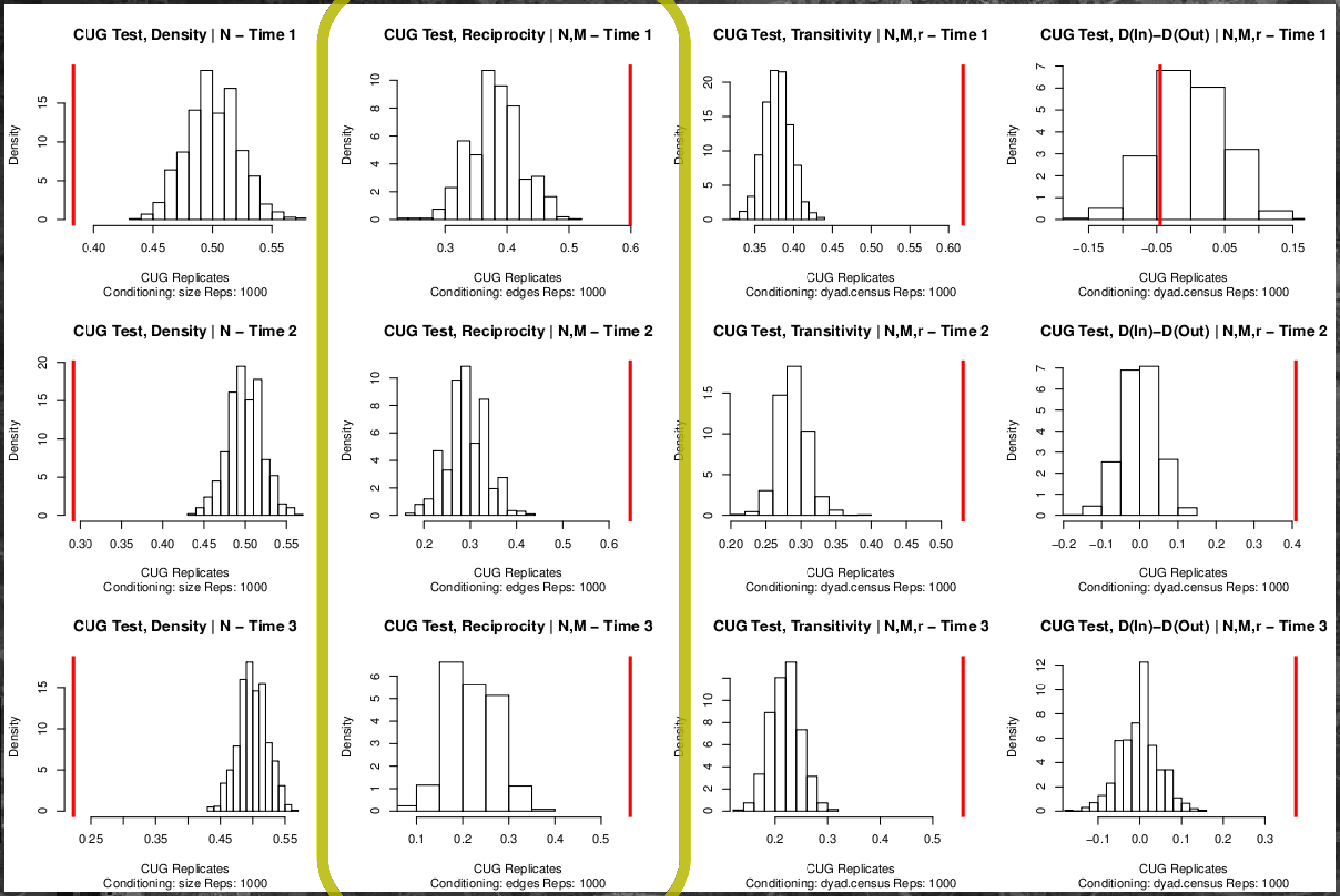

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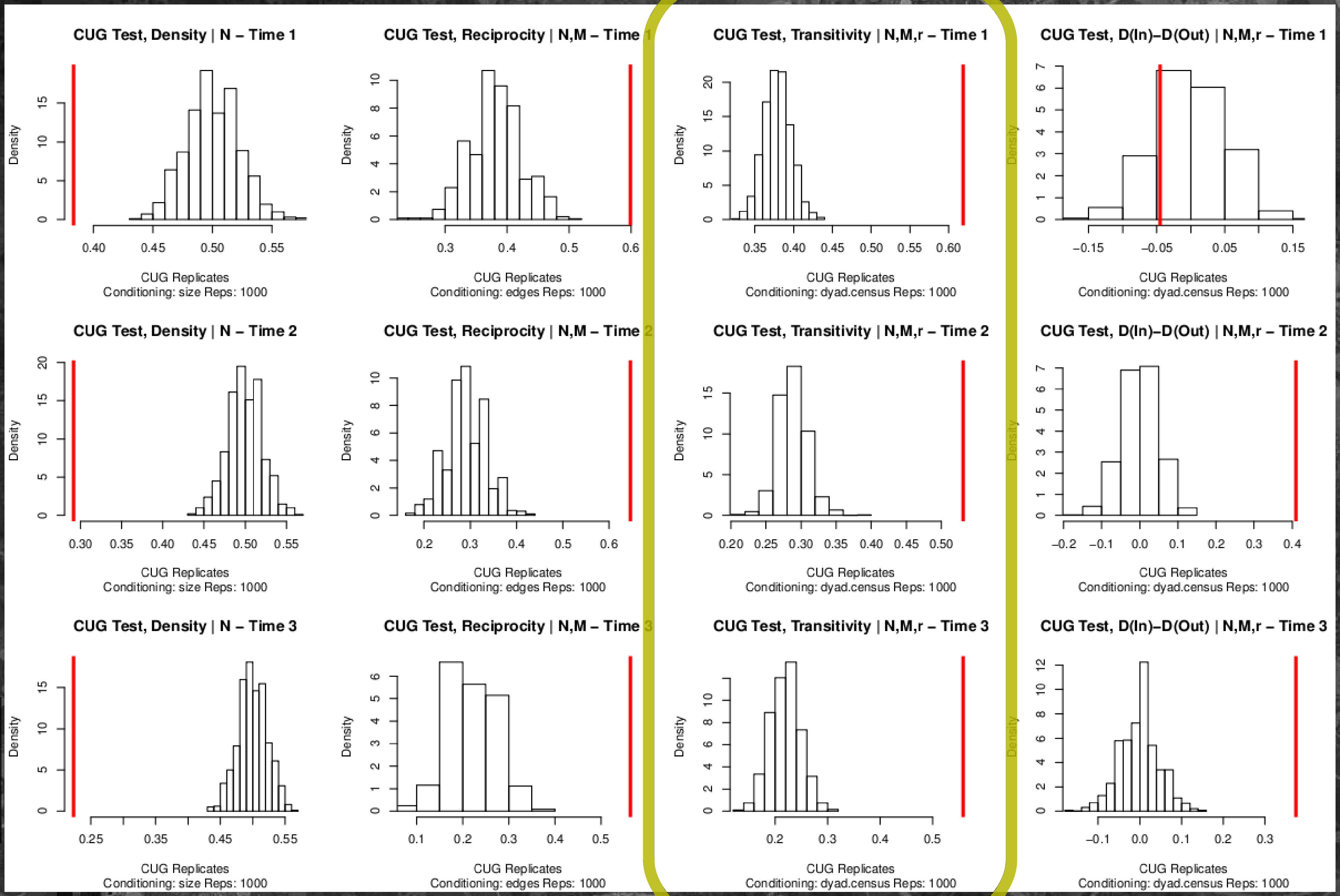

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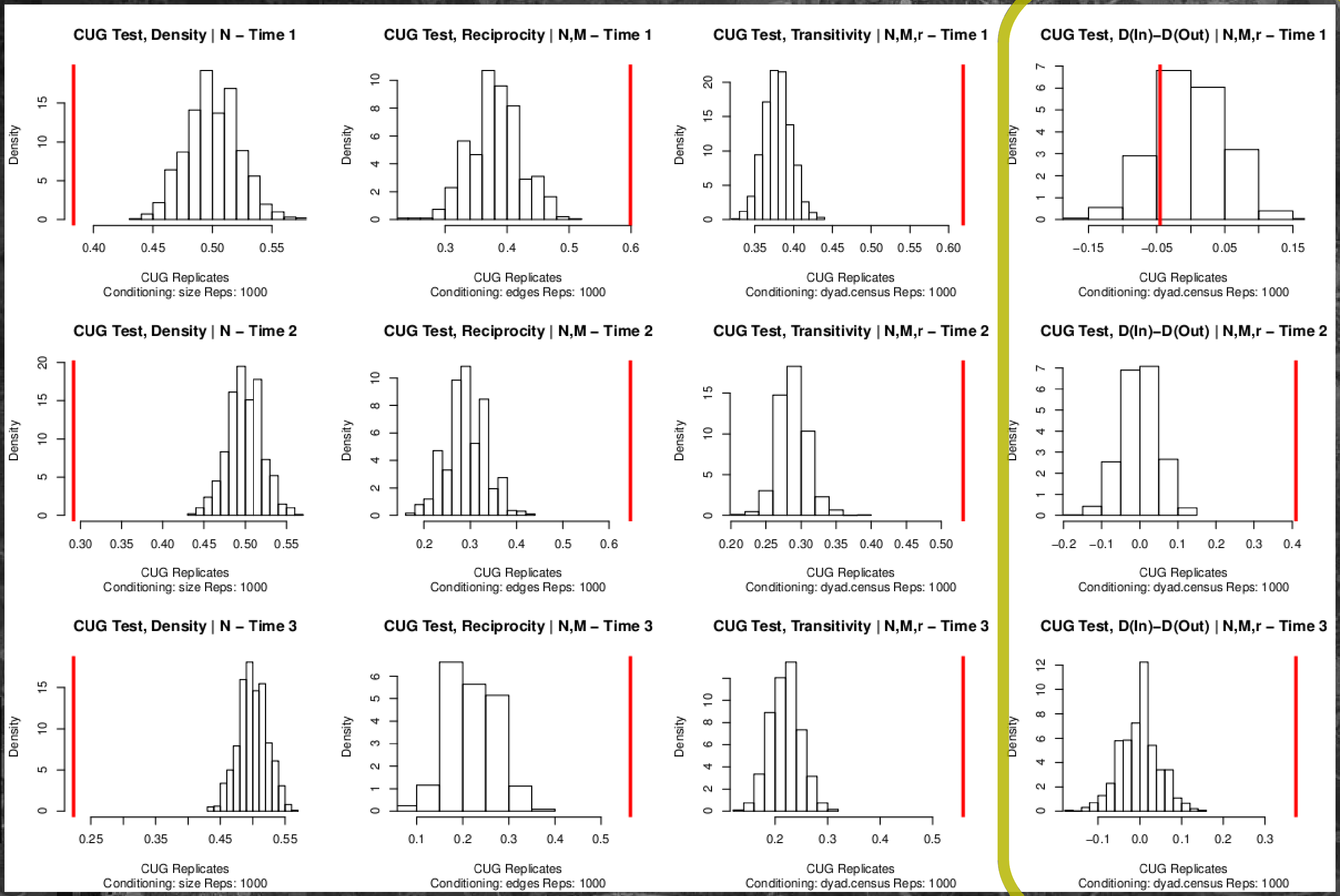
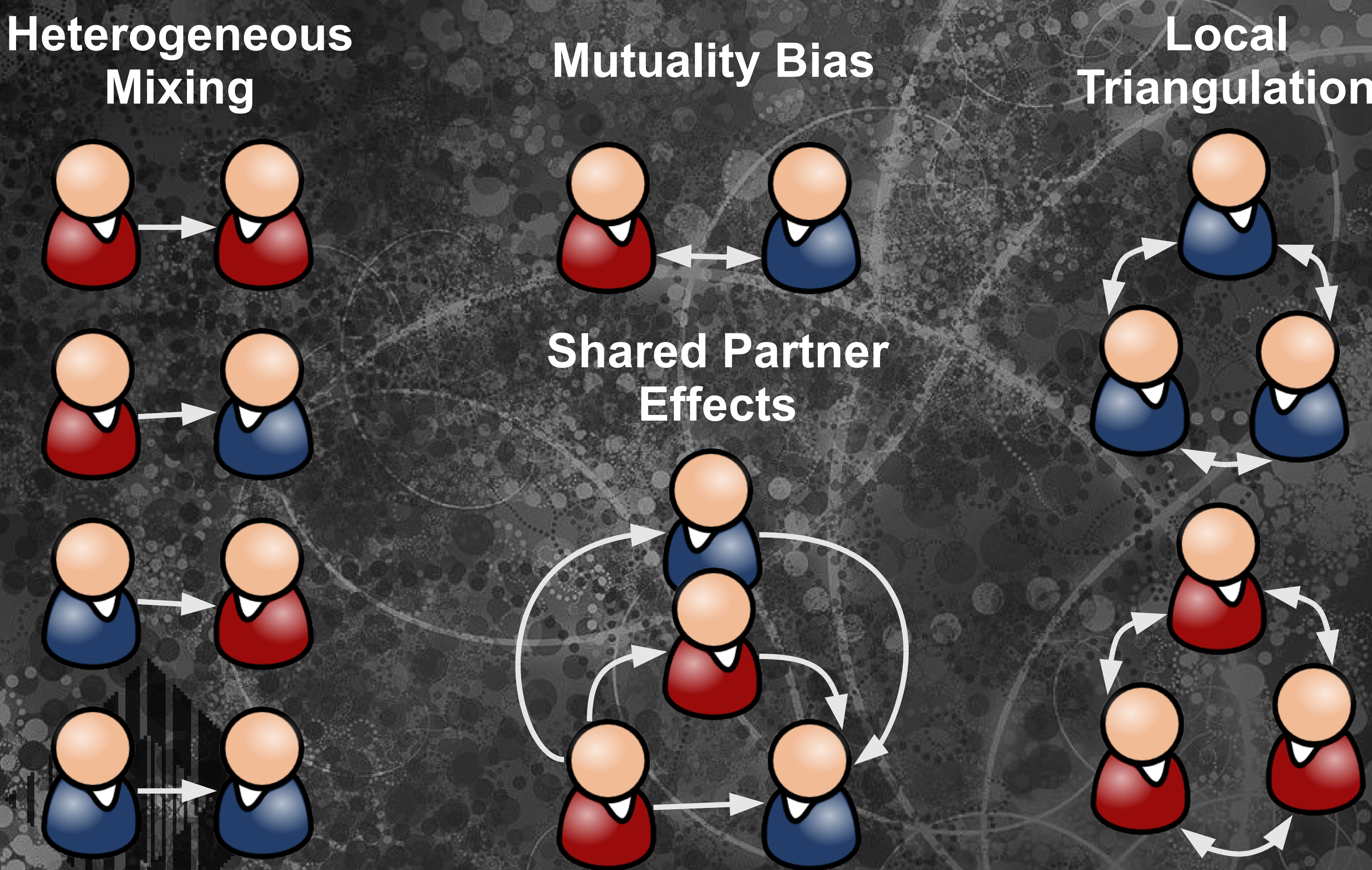


Beyond the Baselines

- Baseline models only take us so far
  - Few statistics lend themselves to conditioning
  - Difficult to look at multiple biases at once
  - Answers are qualitative in nature
  - Hard to account for sampling, error, etc.
  - Given "rejection" of the baseline, no clear path for further modeling

- Solution: parametric models
  - Identify candidate structural mechanisms
  - Parameterize using graph statistics
  - Fit models to data
    - Compare alternatives
    - Interpret parameter estimates
    - Assess adequacy
  - Can apply/extend for prediction, etc.

Sample Mechanisms



Evaluating Competing Explanations

Edges	Mixing	Mutuals	GWESP	LocalTri	AIC	Rank
1	0	0	0	0	1777.684	15
1	1	0	0	0	1565.073	14
1	0	1	0	0	1516.578	13
1	0	0	1	0	1227.656	2
1	0	0	0	1	1478.532	12
1	1	1	0	0	1428.158	11
1	1	0	1	0	1279.456	6
1	1	0	0	1	1416.441	10
1	0	1	1	0	1234.932	3
1	0	1	0	1	1348.794	9
1	0	0	1	1	1290.241	7
1	1	1	1	0	1216.762	1
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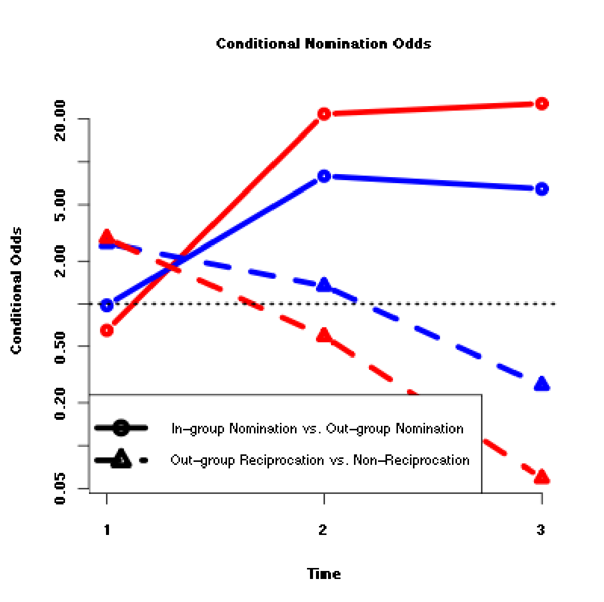
Interpreting the Mechanisms

	Time 1 MLE(SE)	Time 2 MLE(SE)	Time 3 MLE(SE)
Red→Red	-1.853 (0.291)	0.557 (0.226)	-1.069 (0.363)
Red→Blue	-1.421 (0.277)	-2.521 (0.428)	-4.317 (0.752)
Blue→Red	-1.501 (0.286)	-1.705 (0.354)	-2.809 (0.417)
Blue→Blue	-1.527 (0.198)	0.364 (0.226)	-0.948 (0.269)
Mutuals	2.484 (0.328)	1.992 (0.335)	1.489 (0.399)
GWESP	-0.030 (0.019)	-0.427 (0.031)	-0.018 (0.104)
GWESP(α)	1.218 (1.248)	0.744 (0.111)	0.598 (6.572)

- Sharp decline in out-group nomination propensity w/out systematic in-group shift
  - Conditional odds of in-group vs out-group nomination increase at time 2, stabilize
  - Effect somewhat stronger for Reds than Blues
- Decline in mutuality
  - Initially, both groups willing to conditionally reciprocate; by time 3, neither is!
- No clear trend in third-party effects
- Overall: out-group prefs, reciprocity key



And Beyond...

- Given an initial model family, there is much more one can do
  - Assess model adequacy versus target descriptives
  - Prediction (conditional, forecasting, scenario evaluation, etc.)
  - Extension/expansion given new data
- These are difficult or impossible using a purely descriptive framework (or even baseline models)

Looking Ahead

- Today's talks and posters will expand on these themes in various ways....
  - New methods for fitting network models
  - Algorithms to improve performance
  - New ways of parameterizing models
  - Applications to complex data sets
  - (and more!)
- Lots of work is in progress - don't hesitate to ask!