

Exponential random graphs and dynamic graph algorithms

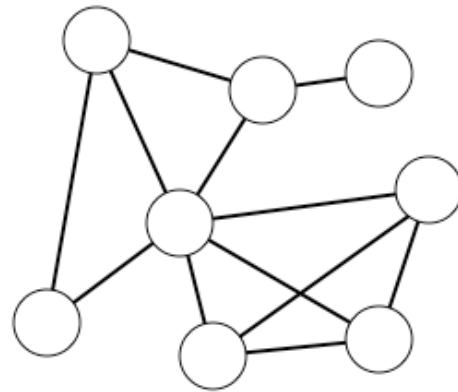


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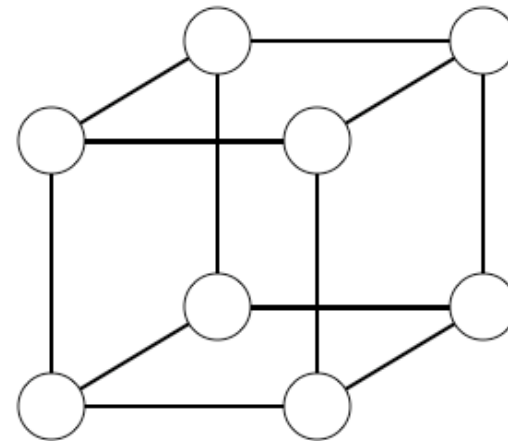
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What are we trying to do?

- Probabilistic reasoning on social networks
- Appropriately model the different likelihoods of finding different types of network



Likely



Unlikely



Exponential Random Graphs

- Family of graphs with fixed vertex set
- Probability of a graph is proportional to $\exp(\text{sum of weights of features})$
- Different choices of features give simpler or more powerful models



ERG models can be simple...

- Easily subsumes many standard random graph models
- E.g. $G(n,p)$:
 - Edges are independent w/probability p
 - Feature = edge
 - Weight = $\log(p) - \log(1-p)$



...but ERG models can also be very powerful

- Powerful enough to represent any distribution over n -vertex graphs
 - Feature = isomorphism with one graph
 - Weight = $\log(\text{probability of that graph})$
- More power requires a more complex set of features



Computational tasks for reasoning with ERGs

- Compute normalizing factor (partition function) for graph probabilities
- Generate random graphs from the model
- Use the model as a prior for maximum likelihood data fitting, or modify the feature weights to fit the data



Monte Carlo methods for computing with ERGs

- Start with an arbitrary graph
- Repeatedly propose a small change (e.g., insert or delete a single edge)
- Compute log-likelihood of the modified graph and use it to accept or reject the proposed change



The Algorithmic Lens

- Social scientists and statisticians determine the sorts of models that best describe their data
- Algorithms researchers (e.g. me) figure out how to make the model run quickly
- Faster algorithms lead to the ability to use more accurate models



Algorithmic rephrasing of the computational task

- Maintain a dynamic graph subject to edge insertions and deletions
- As the graph changes, keep track of its computational properties efficiently (faster than recomputing them from scratch)
- The properties we track should be the ones needed for ERG feature vectors



A brief survey of dynamic graph algorithms

- Sparsification (E., Galil, Italiano, Nissenzweig, JACM '92):
 - Replace dense graphs by tree of sparse subgraphs
 - Applies to many problems including maintaining connected components
 - Replaces #edges by #vertices in running times of update algorithms



A brief survey of dynamic graph algorithms

- Fast dynamic connectivity (Holm, de Lichtenberg, Thorup, JACM 2001):
 - Maintain connected components, number of connected components, or a spanning tree (so can use #components as ERG feature)
 - Update time $O(\log n \log \log n)$
 - Complicated, of interest to search for more easily implemented variants



A brief survey of dynamic graph algorithms

- Distance and reachability in graphs
 - Of likely use in ERGs (e.g. to model small-world properties of these graphs)
 - Some dynamic graph algorithms are known but more theoretical than practical



A brief survey of dynamic graph algorithms

- Graphs in the plane and on surfaces
 - E. et al, J. Algorithms 1992
 - E. et al, J. Comp. Sys. Sci. 1996
 - E., SODA 2002
- Of possible interest for integrating social networks with geographic data



Not-yet-dynamized graph algorithms useful for ERGs

- Low-degree orientations of sparse graphs (Chrobak, E., Theor. Comp. Sci. 1991)
 - Assign directions to the edges of the graph so that each vertex has $O(1)$ outgoing edges
 - Enable fast search for small subgraphs (e.g. list all cliques in linear time)
 - May be found in linear time



Not-yet-dynamized graph algorithms useful for ERGs

- Finding all maximal complete bipartite subgraphs in a sparse graph (E., IPL 1994)
 - Allows concise representation of all four-vertex cycles (quadratically many cycles may be represented in linear space and time)
 - Based on low-degree orientation



Not-yet-dynamized graph algorithms useful for ERGs

- Subgraph isomorphism: finding all copies of some small pattern graph in a larger graph (E., J. Graph Th. 1993 and J. Graph Algorithms 1999)
 - Commonly used as ERG features
 - Known fast algorithms rely on special graph properties e.g. planarity



Conclusions

- ERG are important model for social nets
- ERG computation naturally involves dynamic graphs
- Many existing dynamic graph algorithms known, not fully adapted to ERG problems
- Much opportunity for further study of dynamic graph algorithms in ERG setting