Modeling Relational Event Dynamics

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Overview

- Content in a nutshell
  - Yet Another Framework for modeling social microdynamics
  - Another one? Why?
    - Fairly general
    - Principled basis for inference (estimation, model comparison, etc.) from actually existing data
    - Utilizes well-understood formalisms (event history analysis, discrete exponential families)
    - Fills a gap in current modeling capabilities

- Today:
  - Introduction to modeling approach
  - Sample application to WTC radio conversation (if time allows!)
Conceptual Motivation: Slicing the Temporal Pie

- How should one deal with dynamics of temporally non-extensive relationships?
- Classic logic: take "slices" through the temporal structure
  - Finer slices reveal a more disaggregated view of the network
- Classic problem: how fine should the slices be?

WTC Channel Z, Vertical Transportation
The Limit of Decomposition: Relational Events

Newark Airport Channel 36, CPD

Aggregate Network

Event Structure
Actions and Relational Events

• Action: discrete event in which one entity emits a behavior directed at one or more entities in its environment
  – Useful "atomic unit" of human (or other!) activity
  – Represent formally by relational events

• Relational event: \( a = (i, j, k, t) \)
  – \( i \in S \): "Sender" of event \( a \); \( s(a) = i \)
  – \( j \in R \): "Receiver" of event \( a \); \( r(a) = j \)
  – \( k \in C \): "Action type" ("category") for event \( a \); \( c(a) = k \)
  – \( t \in \mathbb{R} \): "Time" of event \( a \); \( \tau(a) = t \)
Events in Context

- Multiple actions form an event history,
  \[ A_t = \{ a_i : \tau(a_i) \leq t \} \]
  - Take \( a_0 : \tau(a_0) = 0 \) as "null action", \( \tau(a_i) \geq 0 \)
  - Possible actions at \( t \) given by \( A(A_t) \subseteq S \times R \times C \)
    - Forms support for next action
    - Assume here that \( A(A_t) \) finite, constant between actions; may be fixed, but need not be

- Goal: model \( A_t \)
  - Treat actions as events in continuous time
  - Hazards depend upon past history, covariates
Event Model Likelihood: Piecewise Exponential Case

- **Natural simplifying assumption**: actions arise as Poisson process with piecewise constant rates
  - Intuition: hazard of each possible event is *locally* constant, which is constant, given complete event history up to that point
    - Waiting times conditionally exponentially distributed
    - Rates *can* change when events transpire, but not otherwise
      - Compare to related assumption in Cox prop. hazards model

- **Can use to implement event likelihood**
  - Let $M = |At|$, $\tau_i = \tau(a_i)$, w/hazard function $\lambda_{ijk} = \lambda(a_i, A_k, \theta)$; then
  $$p(A_t | \theta) = \prod_{i=1}^{M} \left( \lambda_{a_i, A_{\tau_i-1}, \theta} \prod_{a' \in A_{\tau_i-1}} \exp \left( -\lambda_{a', A_{\tau_i-1}, \theta} [\tau_i - \tau_{i-1}] \right) \right) \prod_{a' \in A_{\tau_i}} \exp \left( -\lambda_{a', A, \theta} [t - \tau_M] \right)$$
The Problem of Uncertain Event Timing

- Likelihood of an event sequence depends on the detailed history
  - Problem: exact timing is generally uncertain for many data sources (e.g., transcripts), though order is known
  - What if we only have (temporally) ordinal data?

- Stochastic process theory to the rescue!
  - Thm: Let $X_1,...,X_n$ be independent exponential r.v. w/rate parameters $\lambda_1,...,\lambda_n$. Then the probability that $x_i=\min\{x_1,...,x_n\}$ is $\lambda_i/(\lambda_1+...+\lambda_n)$.
  - Implication: likelihood of ordinal data is a product of multinomial likelihoods
    - Identifies rate function up to a constant factor
Event Model Likelihood: Ordinal Data Case

• Using the above, we may write the likelihood of an event sequence $A_t$ as follows:

$$p(A_t|\theta) = \prod_{i=1}^{M} \left[ \frac{\lambda_{a_t A_{t-1}, \theta}}{\sum_{a' \in A(A_t)} \lambda_{a_t A_{t-1}, \theta}} \right]$$

• Dynamics governed by rate function, $\lambda$

$$\lambda_{a A_t, \theta} = \begin{cases} 
\exp \left( \lambda_0 + \theta^T u \left( s(a), r(a), c(a), A_t, X_a \right) \right) & a \in A(A_t) \\
0 & a \notin A(A_t) 
\end{cases}$$

- Where $\lambda_0$ is an arbitrary constant, $\theta \in \mathbb{R}^p$ is a parameter vector, and $u: (i,j,A_t,X) \rightarrow \mathbb{R}^p$ is a vector of sufficient statistics
Fitting the Event Model

- Given $A_t$ and $u$, how do we estimate $\theta$?
  - Parameters interpretable as logged rate multipliers (in $u$)
- We have $p(A_t|\theta)$, so can conduct likelihood-based inference
  - Find MLE $\theta^* = \arg\max_{\theta} p(A_t|\theta)$, e.g., using a variant Newton-Rapheson or other method
  - Can also proceed in a Bayesian manner
    - Posit $p(\theta)$, work with $p(\theta|A_t) \propto p(A_t|\theta)p(\theta)$
  - Some computational challenges when $|A|$ is large; tricks like MC quadrature needed to deal with sum of rates across support
Example: Relational Dynamics in WTC Communications

- **Data**: six transcripts of radio communications among WTC responders
  - PATH radio communications; Newark police, airport maintenance, and command post radio; NJ SPEN 2; and WTC police
    - Each documents all radio contact within one group
- **Propose effects, fit using ML**
  - Effects based on cognitive, interactional mechanisms
  - Approximate asymptotic standard errors, \( p \)-values using inverse of estimated information matrix at MLE
  - Use BIC to compare models, assess effects
Mechanisms and Effects

- To model conversation dynamics, choose sufficient statistics, $u$, based on prior theory
  - Should incorporate behaviorally meaningful mechanisms, baseline effects

- A first cut: six classes of effects
  - Persistence (P): previous out-alters salient for ego's out-calls
  - Recency (R): more recent in-alters salient for ego's out-calls
  - Triad effects (T): ego tends to seek/avoid out-calls based on transitive/cyclic completions, shared in/out partners
  - Participation shifts (PS): tendencies reflecting "local" conversational norms (from Gibson, 2003)
  - Preferential attachment (PA): ego tends to call those with more airtime
  - Fixed effects (FE): heterogeneity in ego's tendency to send/receive
# Joint and Marginal Models: BIC Scores

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Null    | 927.93                     | 985.13                       | 1048.05                    | 1930.14                  | 4138.34                 | 6812.60                 |

P       | 755.99                     | 702.57                       | 786.26                     | 1684.74                  | 3796.24                 | 5754.44                 |

R       | 659.36                     | 521.08                       | 650.49                     | 1431.95                  | 2946.52                 | 4081.38                 |

T       | 941.95                     | 999.79                       | 1060.45                    | 1780.55                  | 4034.06                 | 5853.89                 |

PS      | 512.57                     | 309.80                       | 361.36                     | 1115.52                  | 2001.39                 | 2493.83                 |

PA      | 902.86                     | 901.04                       | 1021.68                    | 1711.58                  | 3766.50                 | 5703.66                 |

FE      | 920.27                     | 902.53                       | 1041.14                    | 1381.78                  | 3337.86                 | 4308.54                 |

P+R+T+PS| 517.00                     | 331.57                       | 379.95                     | 1040.60                  | 1955.18                 | 2289.74                 |

P+R+T+PS+PA | 520.64                     | 333.54                       | 379.57                     | 1041.73                  | 1946.23                 | 2245.71                 |

P+R+T+PS+FE | 607.69                     | 419.13                       | 470.36                     | 1008.54                  | 2009.70                 | 2308.08                 |

P+R+T+PS+PA+FE | 610.71                    | 423.47                       | 469.99                     | 1011.26                  | 2014.76                 | 2313.65                 |
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P-Shifts provide best marginal models, with Recency a distant second; Fixed Effects important for larger transcripts.
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Minimal joint models with P-Shifts do well, but other effects contribute in large transcripts.
MLEs for Event Model Parameters, w/Asymptotic 95% CIs

Parameter Estimates, Joint Model

Conversational Continuity

Reciprocity (Turn-Taking)

Reciprocity (Mnemonic)

Preferential Attachment (Negative!)

Persistence

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Conclusion

● Relational event model
  – Fairly general form for discrete events with complex historical dependence
    • Can specify event rates in terms of past history, covariates
    • Set of possible events can evolve endogenously
  – Applicable to sequence data as well as complete event histories (although pacing information is lost)

● Sample application; WTC radio communication
  – Clear effects for conversational norms (continuity and turn-taking), recency, persistence, and partner cycling
  – Triadic effects weak to nonexistent
    • Few opportunities in smaller data sets, so high uncertainty
Additional Slides
Explaining Hub Formation

- Two obvious classes of explanation
  - Preferential attachment
    - Exposure effects
    - Emergent specialization
  - Heterogeneity in base activity levels
    - Institutional role
    - The “latent safety” hypothesis
- Modeling the mechanisms
  - Past total degree effect
  - Fixed effects for communication activity
Persistence Effects

- Inertia-like effect: past contacts may tend to become future contacts
  - Unobserved relational heterogeneity
  - Availability to memory
  - (Compare w/autocorrelation terms in an AR process)

- Simple implementation: fraction of previous contacts as predictor
  - Log-rate of $(i,j)$ contact adjusted by $\theta d_{ij}/d_i$
Recency/Ordering Effects

• Ordering of past contact potentially affects future contact
  - Reciprocity norms
  - Recency effects (salience)

• Simple parameterization: dyadic contact ordering effect
  - Previous incoming contacts ranked
    • Non-contacts treated as rank \( \infty \)
  - Log-rate of outgoing \((i,j)\) contact adjusted by \(\theta(1/rank_{ji})\)
Can also control for endogenous triadic mechanisms

- Two-path effects
  - Past outbound two-path flows lead to/inhibit direct contact (transitivity)
  - Past inbound two-path flows lead to/inhibit direct contact (cyclicity)

- Shared partner effects
  - Past outbound shared partners lead to/inhibit direct contact (common reference)
  - Past inbound shared partners lead to/inhibit direct contact (common contact)
Coordination, Hub Status and Institutional Role

- Importance of coordination well-known among practitioners (e.g., Auf der Heide, 1989)
  - Response organizations include institutionalized coordinative roles, e.g., dispatchers, call desk operators
- Can hub status be explained via existing roles?
  - “Institutionalized” vs. “emergent” coordinators (a la Dynes, 1970)
- Coding from transcript content
  - Title includes “command,” “desk,” “operator,” “dispatch,” “manager,” “control,” or “base”
  - Responder identified with site (“Newark Airport”)