Modeling Relational Event Dynamics

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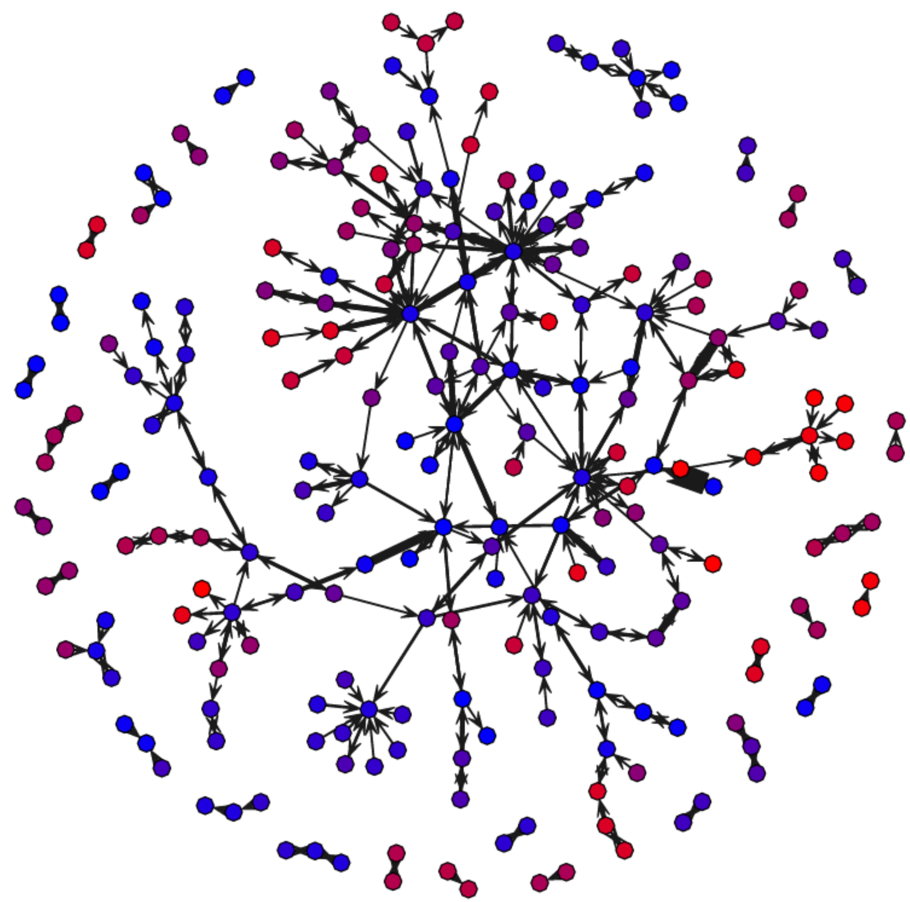
Overview

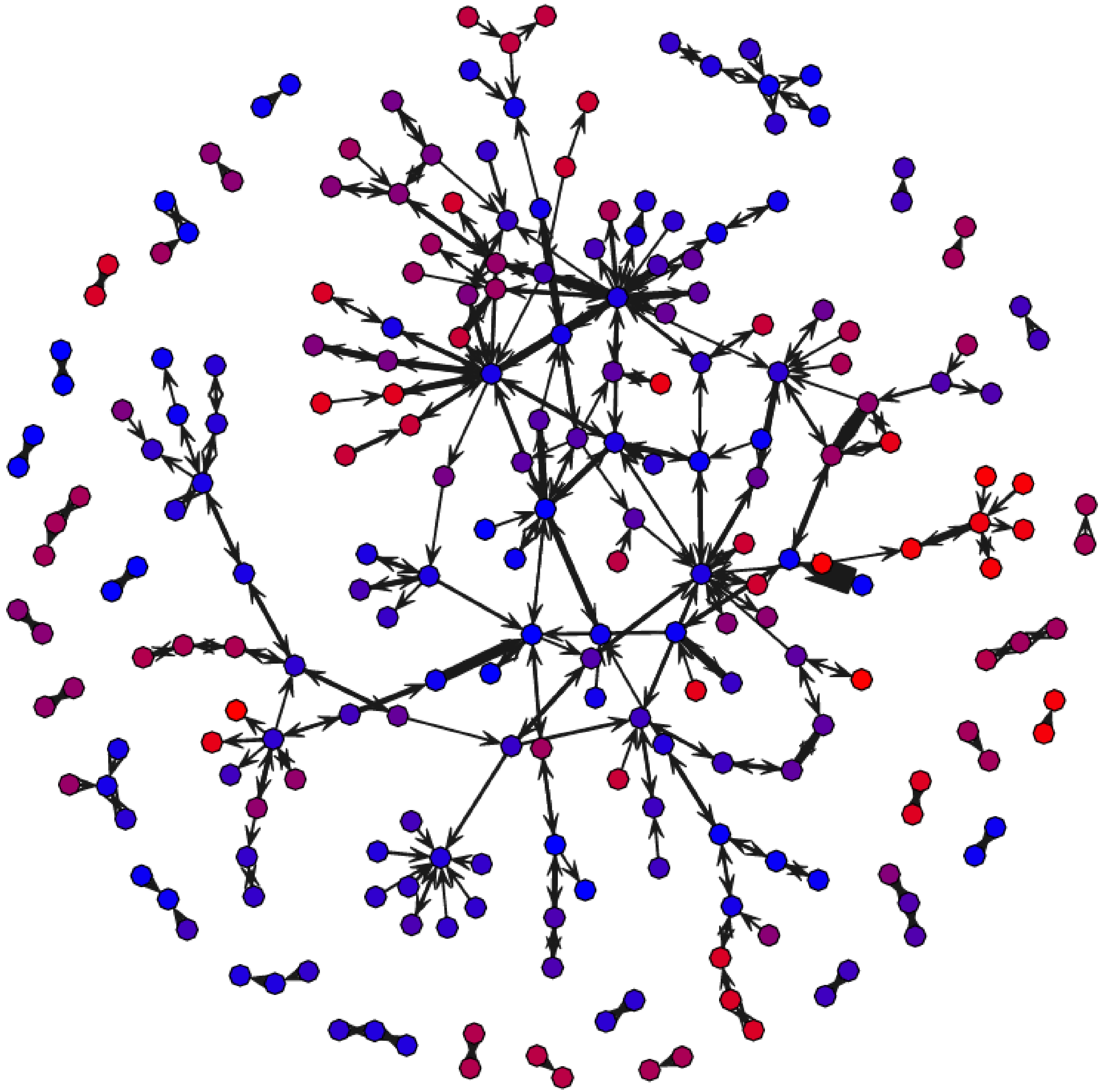
- **Content in a nutshell**
  - Yet Another Framework for modeling social microdynamics
  - Another one? Why?
    - Fairly general
    - Principled basis for inference (estimation, model comparison, etc.) from actually existing data
    - Utilizes well-understood formalisms (event history analysis, discrete exponential families)
    - Fills a gap in current modeling capabilities
- **Today:**
  - Introduction to modeling approach
  - Sample application to WTC radio conversation (if time allows!)

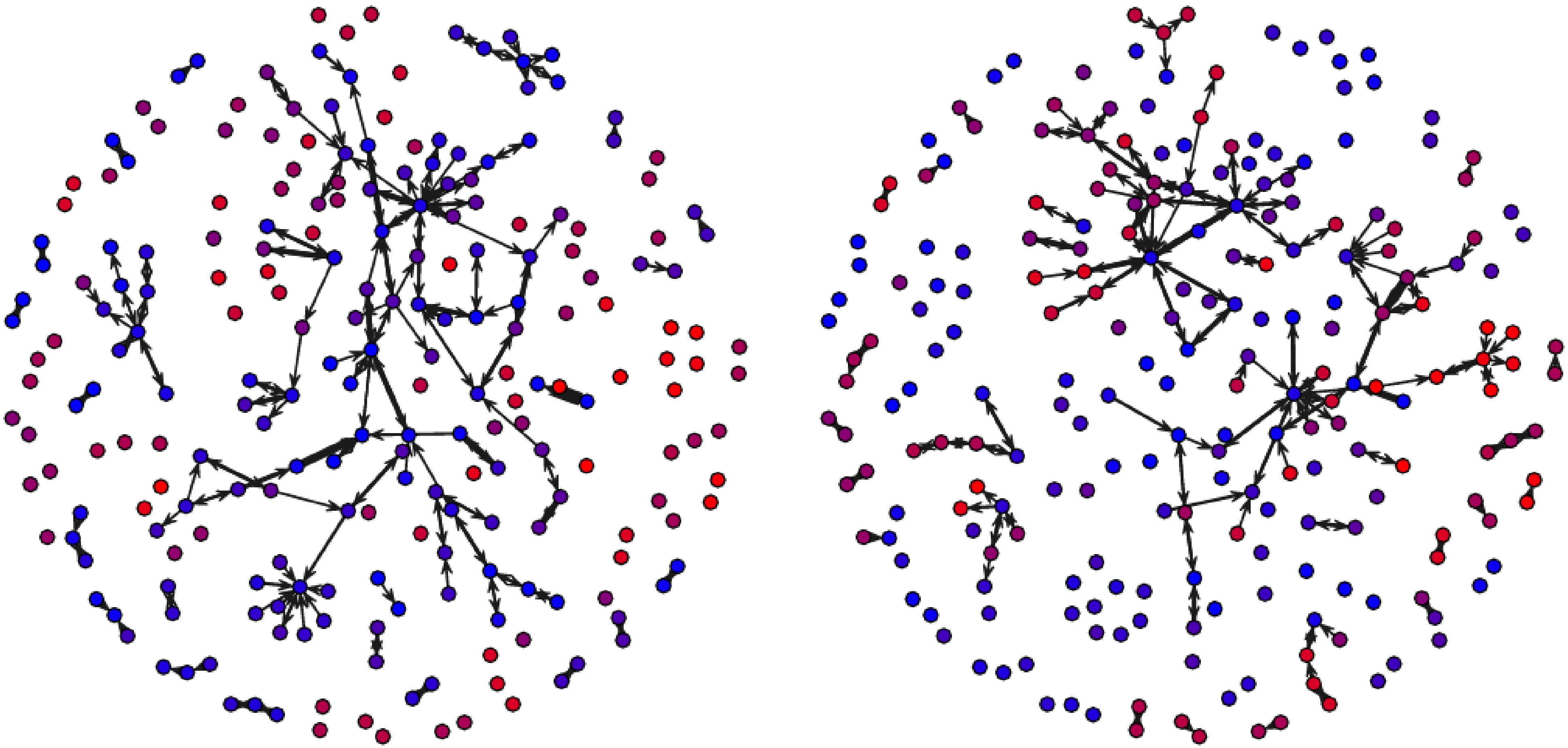
Conceptual Motivation: Slicing the Temporal Pie

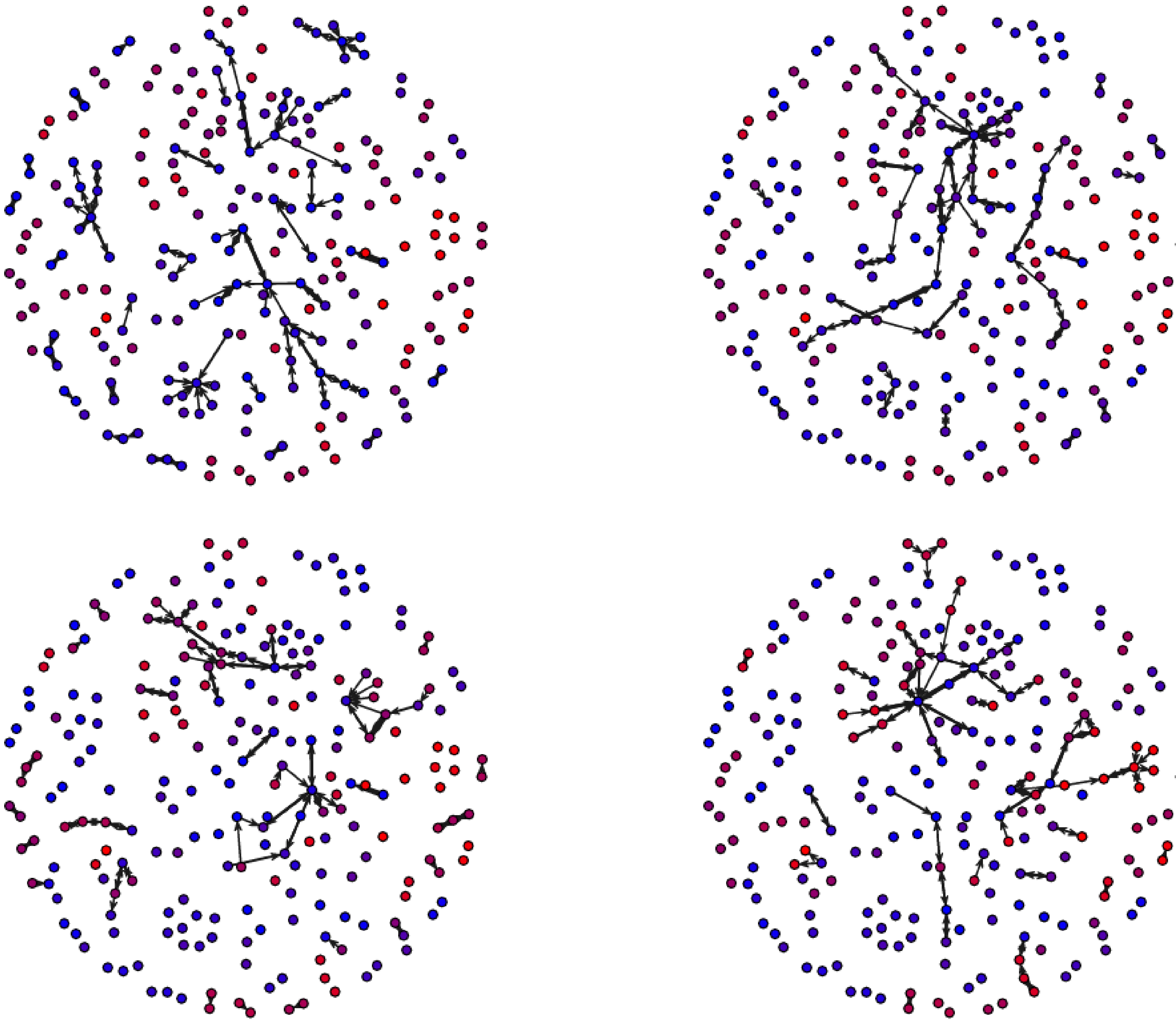
- **How should one deal with dynamics of temporally non-extensive relationships?**
- **Classic logic: take "slices" through the temporal structure**
  - Finer slices reveal a more disaggregated view of the network
- **Classic problem: how fine should the slices be?**

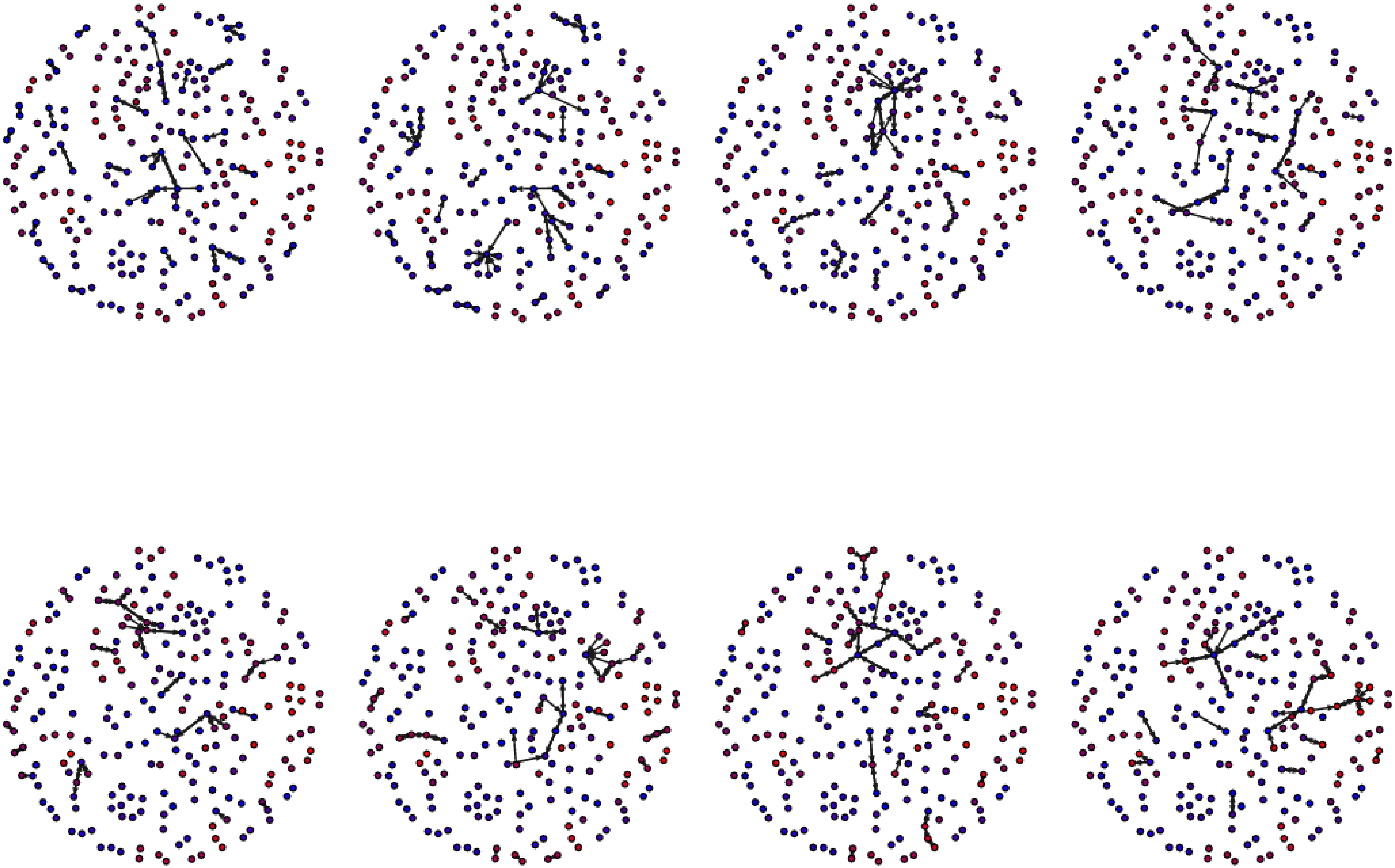


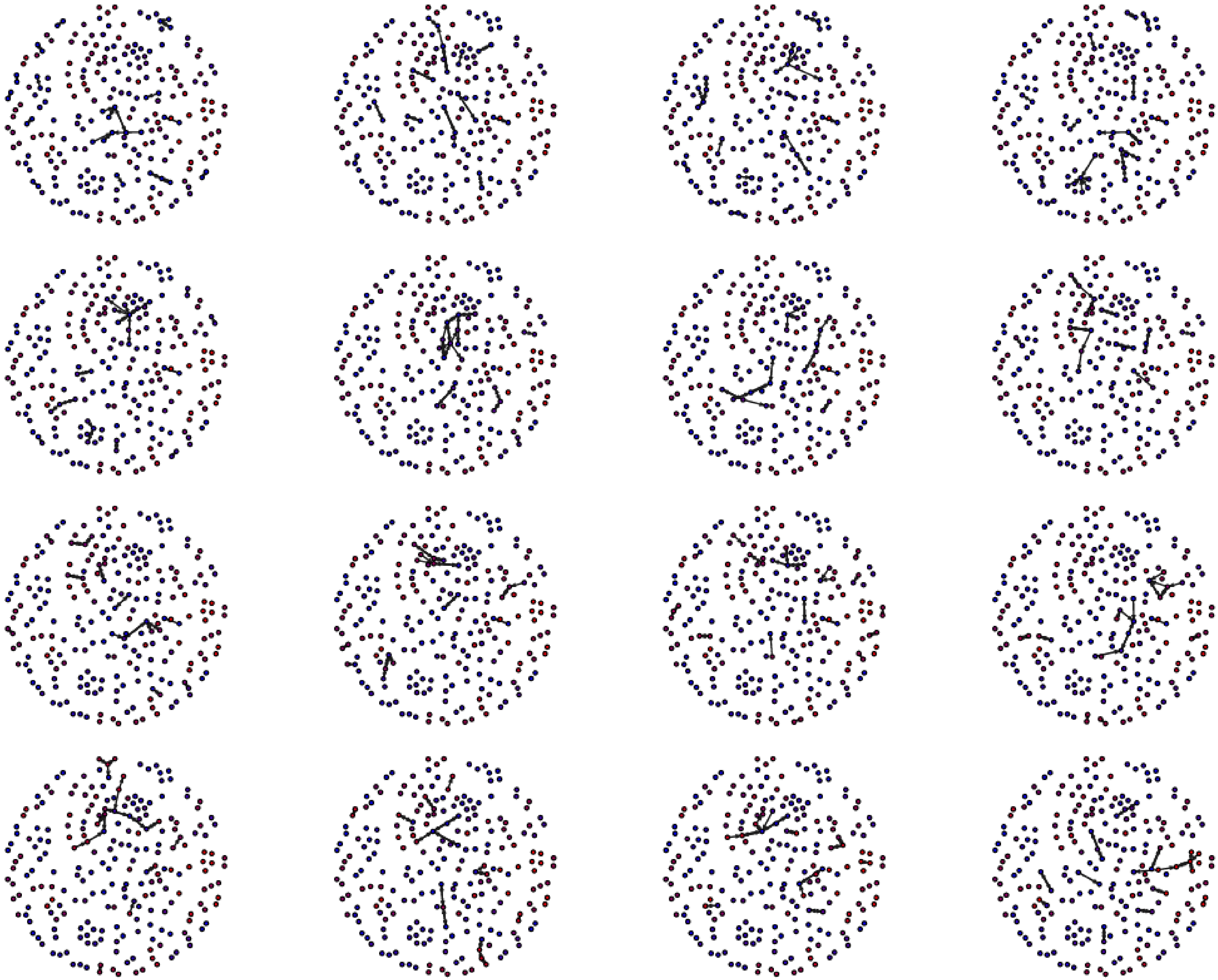
WTC Channel Z, Vertical Transportation





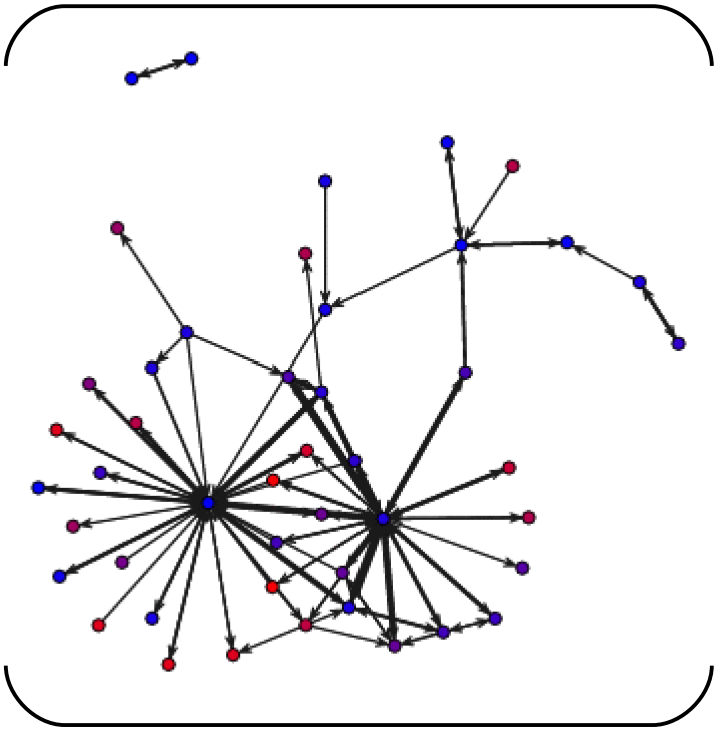




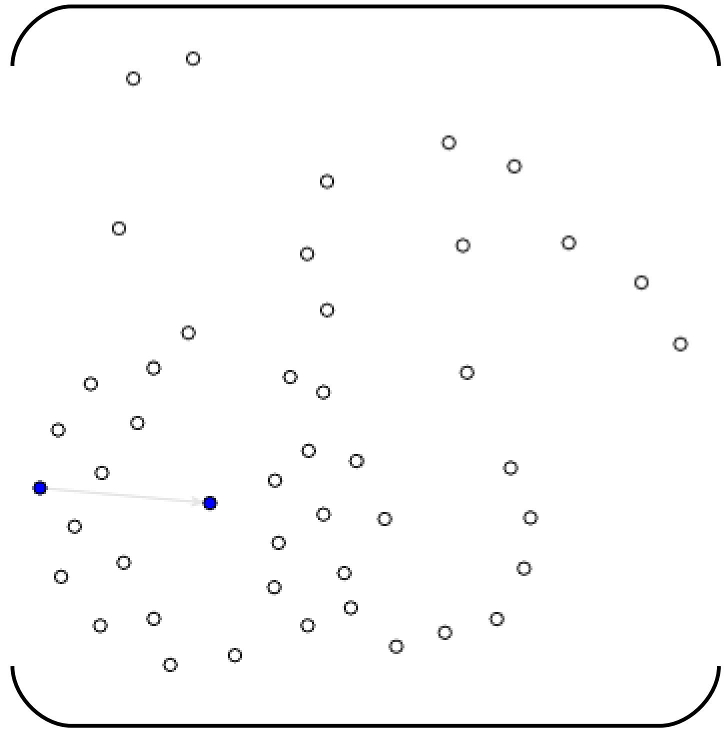


The Limit of Decomposition: Relational Events

Newark Airport Channel 36, CPD



Aggregate Network



Event Structure

(MPG)

Actions and Relational Events

- **Action: discrete event in which one entity emits a behavior directed at one or more entities in its environment**
  - Useful "atomic unit" of human (or other!) activity
  - Represent formally by *relational events*
- **Relational event:** $a=(i,j,k,t)$
  - $i \in \mathcal{S}$: "Sender" of event a; $s(a)=i$
  - $j \in \mathcal{R}$: "Receiver" of event a; $r(a)=j$
  - $k \in \mathcal{C}$: "Action type" ("category") for event a; $c(a)=k$
  - $t \in \mathrm{R}$: "Time" of event a; $\tau(a)=t$

Events in Context

- **Multiple actions form an *event history,*** $A_t = \{a_i : \tau(a_i) \leq t\}$
  - Take $a_0 : \tau(a_0) = 0$ as "null action", $\tau(a_i) \geq 0$
  - Possible actions at t given by A$(A_t) \subseteq \mathcal{S} \times \mathcal{R} \times \mathcal{C}$
    - Forms support for next action
    - Assume here that A(A_t) finite, constant between actions; may be fixed, but need not be
- **Goal: model** A_t
  - Treat actions as events in continuous time
  - Hazards depend upon past history, covariates

Event Model Likelihood: Piecewise Exponential Case

- **Natural simplifying assumption: actions arise as Poisson process with piecewise constant rates**
  - Intuition: hazard of each possible event is *locally* constant, which is constant, given complete event history up to that point
    - Waiting times conditionally exponentially distributed
    - Rates *can* change when events transpire, but not otherwise
      - Compare to related assumption in Cox prop. hazards model
- **Can use to implement event likelihood**
  - Let $M=|At|$, $\tau_i=\tau(a_i)$, w/hazard function $\lambda_{ijk}=\lambda(a_i,A_k,\theta)$; then

$$p(A_t|\theta)=\left[\prod_{i=1}^{M}\left(\lambda_{a_i A_{\tau_{i-1}}\theta}\prod_{a'\in \mathrm{A}(A_{\tau_i})}\exp\left(-\lambda_{a' A_{\tau_{i-1}}\theta}\left[\tau_i-\tau_{i-1}\right]\right)\right)\right]\left[\prod_{a'\in \mathrm{A}(A_t)}\exp\left(-\lambda_{a' A_t\theta}\left[t-\tau_M\right]\right)\right]$$

The Problem of Uncertain Event Timing

- **Likelihood of an event sequence depends on the detailed history**
  - Problem: exact timing is generally uncertain for many data sources (e.g., transcripts), though order is known
  - What if we only have (temporally) ordinal data?
- **Stochastic process theory to the rescue!**
  - Thm: Let $X_1,\ldots,X_n$ be independent exponential r.v. w/rate parameters $\lambda_1,\ldots,\lambda_n$. Then the probability that $x_i=\min\{x_1,\ldots,x_n\}$ is $\lambda_i/(\lambda_1+\ldots+\lambda_n)$.
  - Implication: likelihood of ordinal data is a product of multinomial likelihoods
    - Identifies rate function up to a constant factor

Event Model Likelihood: Ordinal Data Case

- **Using the above, we may write the likelihood of an event sequence A_t as follows:**

$$p(A_t|\theta)=\prod_{i=1}^{M}\left[\frac{\lambda_{a_i A_{\tau_{i-1}}\theta}}{\sum_{a'\in \mathrm{A}(A_{\tau_i})}\lambda_{a_i A_{\tau_{i-1}}\theta}}\right]$$

- **Dynamics governed by rate function, λ**

$$\lambda_{a A_t \theta}=\begin{cases}\exp\left(\lambda_0+\theta^T u\left(s(a), r(a), c(a), A_t, X_a\right)\right) & a\in \mathrm{A}(A_t)\\ 0 & a\notin \mathrm{A}(A_t)\end{cases}$$

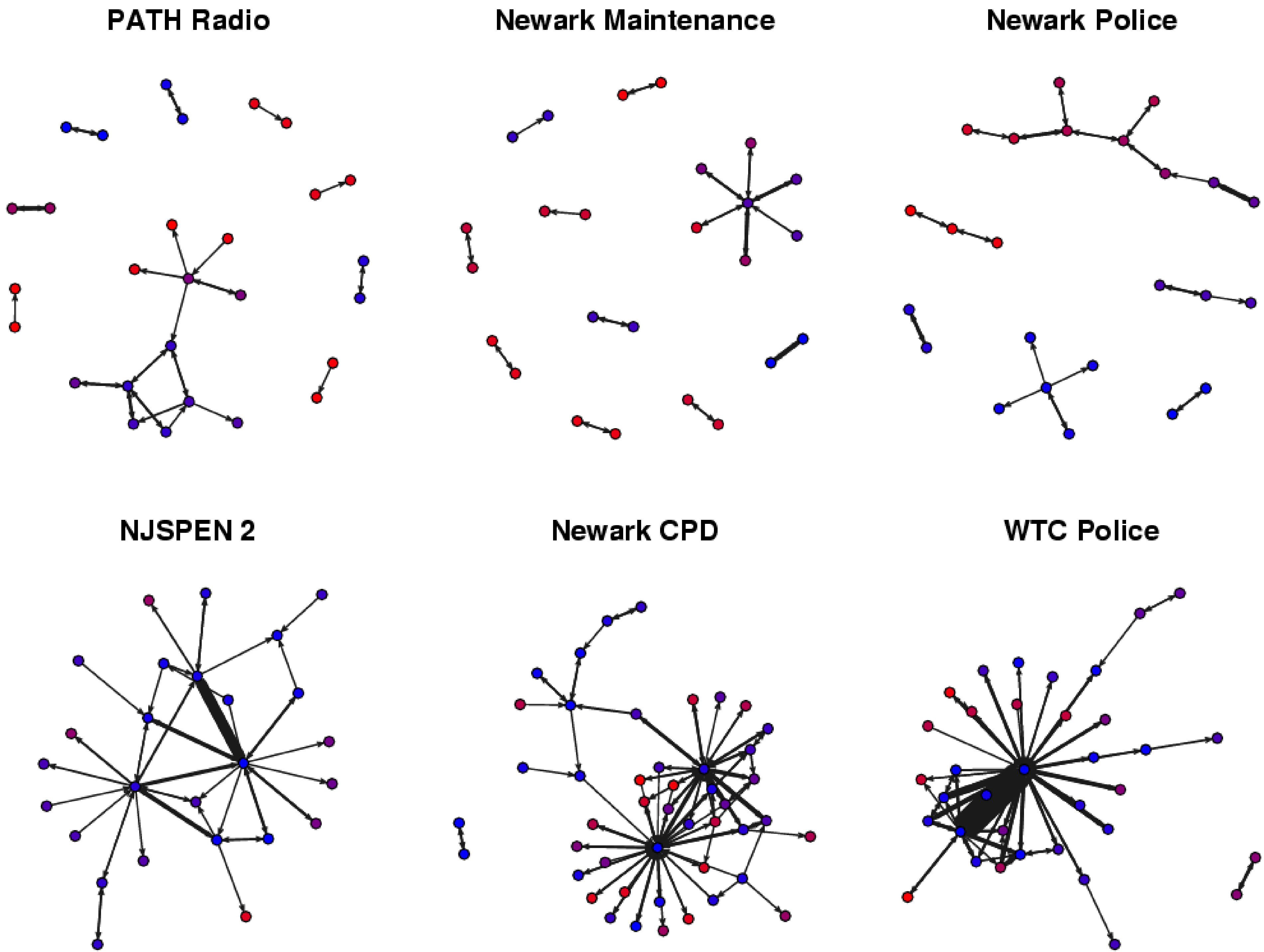
  - Where λ_0 is an arbitrary constant, $\theta \in \mathbb{R}^p$ is a parameter vector, and u: $(i,j,A_t,X) \rightarrow \mathbb{R}^p$ is a vector of sufficient statistics

Fitting the Event Model

- **Given A_t and u, how do we estimate θ?**
  - Parameters interpretable as logged rate multipliers (in u)
- **We have $p(A_t|\theta)$, so can conduct likelihood-based inference**
  - Find MLE $\theta^* = \arg\max_\theta p(A_t|\theta)$, e.g., using a variant Newton-Rapheson or other method
  - Can also proceed in a Bayesian manner
    - Posit $p(\theta)$, work with $p(\theta|A_t) \propto p(A_t|\theta)p(\theta)$
  - Some computational challenges when |A| is large; tricks like MC quadrature needed to deal with sum of rates across support

Example: Relational Dynamics in WTC Communications

- **Data: six transcripts of radio communications among WTC responders**
  - PATH radio communications; Newark police, airport maintenance, and command post radio; NJ SPEN 2; and WTC police
    - Each documents all radio contact within one group
- **Propose effects, fit using ML**
  - Effects based on cognitive, interactional mechanisms
  - Approximate asymptotic standard errors, *p*-values using inverse of estimated information matrix at MLE
  - Use BIC to compare models, assess effects


PATH Radio
Newark Maintenance
Newark Police
NJSPEN 2
Newark CPD
WTC Police

Mechanisms and Effects

- **To model conversation dynamics, choose sufficient statistics, u, based on prior theory**
  - Should incorporate behaviorally meaningful mechanisms, baseline effects
- **A first cut: six classes of effects**
  - Persistence (P): previous out-alters salient for ego's out-calls
  - Recency (R): more recent in-alters salient for ego's out-calls
  - Triad effects (T): ego tends to seek/avoid out-calls based on transitive/cyclic completions, shared in/out partners
  - Participation shifts (PS): tendencies reflecting "local" conversational norms (from Gibson, 2003)
  - Preferential attachment (PA): ego tends to call those with more airtime
  - Fixed effects (FE): heterogeneity in ego's tendency to send/receive

Joint and Marginal Models: BIC Scores

Network	PATH Radio	Newark Maint	Newark Police	NJSPEN 2	Newark CPD	WTC Police
N	28	25	24	26	46	35
M	70	77	83	149	271	481
Null	927.93	985.13	1048.05	1930.14	4138.34	6812.60
P	755.99	702.57	786.26	1684.74	3796.24	5754.44
R	659.36	521.08	650.49	1431.95	2946.52	4081.38
T	941.95	999.79	1060.45	1780.55	4034.06	5853.89
PS	512.57	309.80	361.36	1115.52	2001.39	2493.83
PA	902.86	901.04	1021.68	1711.58	3766.50	5703.66
FE	920.27	902.53	1041.14	1381.78	3337.86	4308.54
P+R+T+PS	517.00	331.57	379.95	1040.60	1955.18	2289.74
P+R+T+PS+PA	520.64	333.54	379.57	1041.73	1946.23	2245.71
P+R+T+PS+FE	607.69	419.13	470.36	1008.54	2009.70	2308.08
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P-Shifts provide best marginal models, with Recency a distant second; Fixed Effects important for larger transcripts

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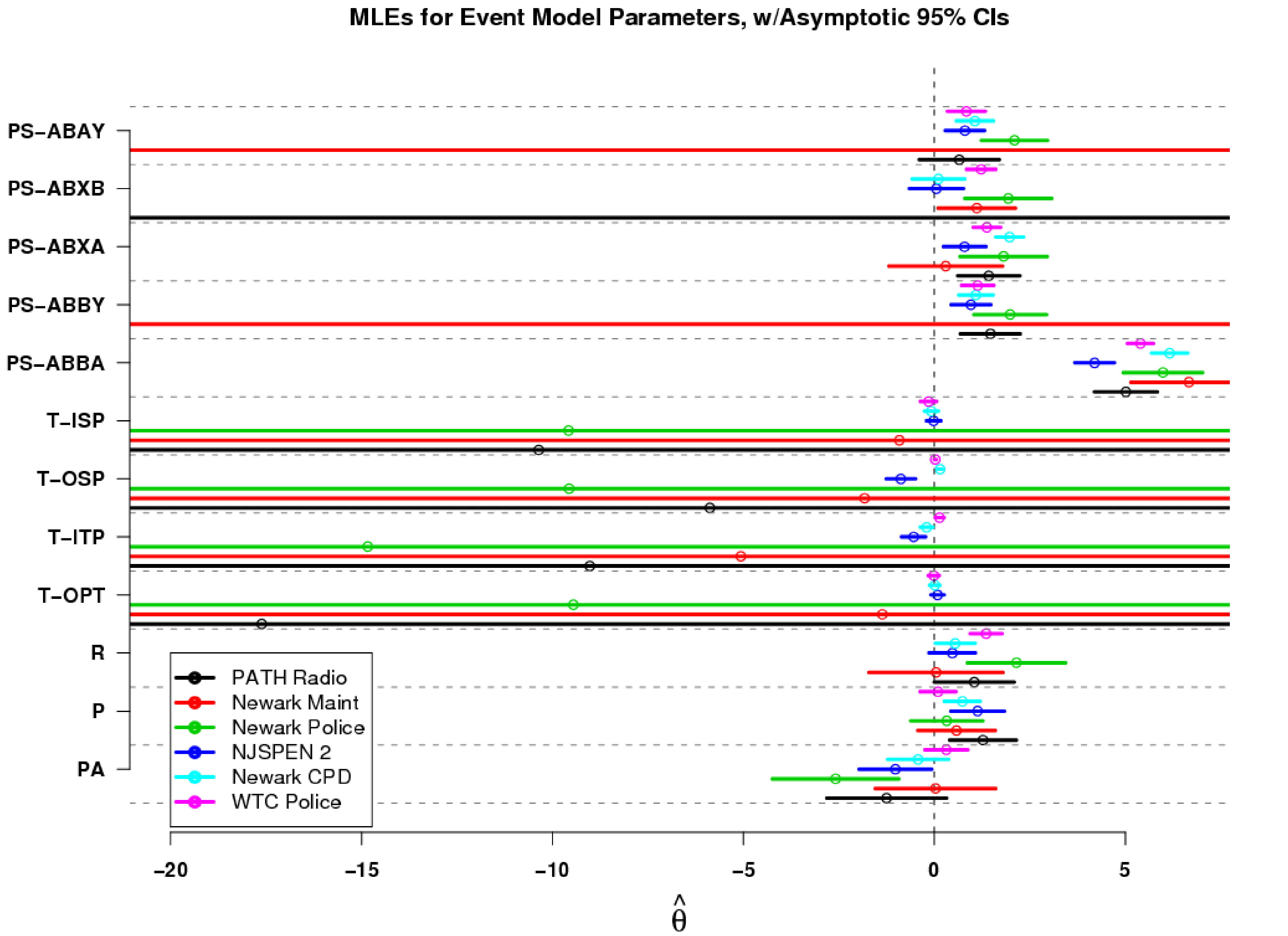
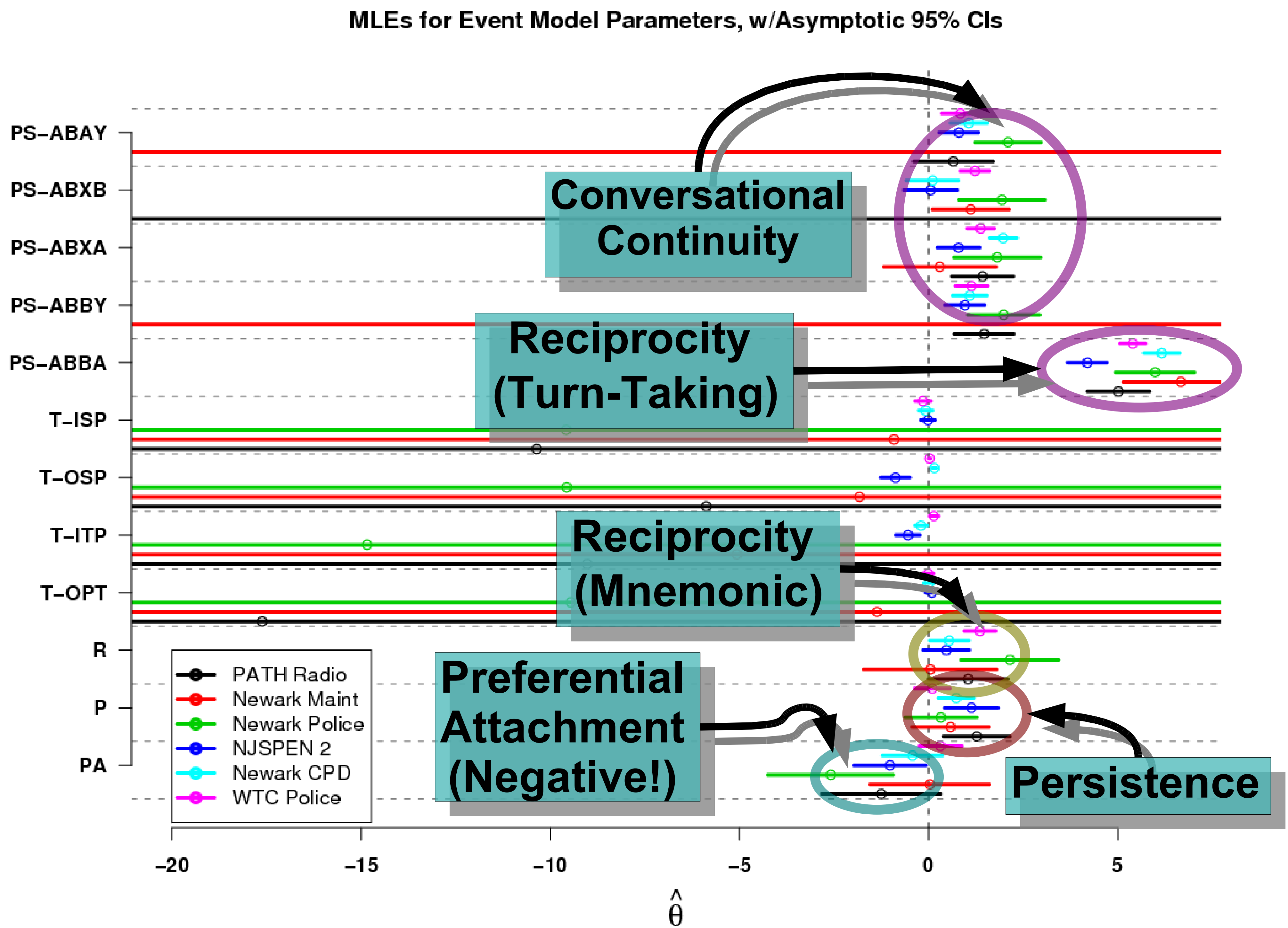
Minimal joint models with P-Shifts do well, but other effects contribute in large transcripts


MLEs for Event Model Parameters, w/Asymptotic 95% CIs
PS−ABAY
PS−ABXB
PS−ABXA
PS−ABBY
PS−ABBA
T−ISP
T−OSP
T−ITP
T−OPT
R
P
PA
PATH Radio
Newark Maint
Newark Police
NJSPEN 2
Newark CPD
WTC Police
−20
−15
−10
−5
0
5
$\hat{\theta}$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θ̂
PATH Radio
Newark Maint
Newark Police
NJSPEN 2
Newark CPD
WTC Police
Conversational Continuity
Reciprocity (Turn-Taking)
Reciprocity (Mnemonic)
Preferential Attachment (Negative!)
Persistence

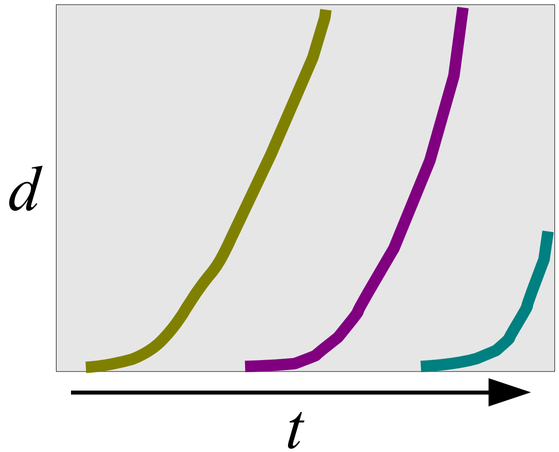
Conclusion

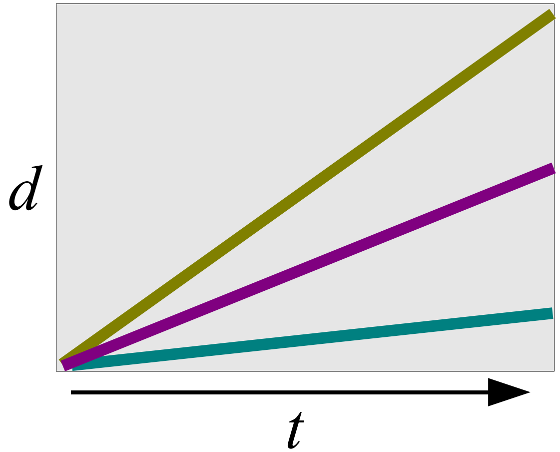
- **Relational event model**
  - Fairly general form for discrete events with complex historical dependence
    - Can specify event rates in terms of past history, covariates
    - Set of possible events can evolve endogenously
  - Applicable to sequence data as well as complete event histories (although pacing information is lost)
- **Sample application; WTC radio communication**
  - Clear effects for conversational norms (continuity and turn-taking), recency, persistence, and partner cycling
  - Triadic effects weak to nonexistent
    - Few opportunities in smaller data sets, so high uncertainty

Additional Slides

Explaining Hub Formation

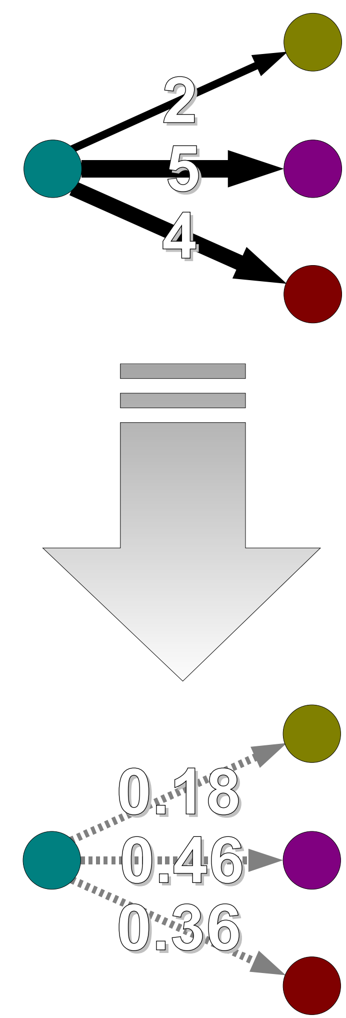
- **Two obvious classes of explanation**
  - Preferential attachment
    - Exposure effects
    - Emergent specialization
  - Heterogeneity in base activity levels
    - Institutional role
    - The “latent safety” hypothesis
- **Modeling the mechanisms**
  - Past total degree effect
  - Fixed effects for communication activity





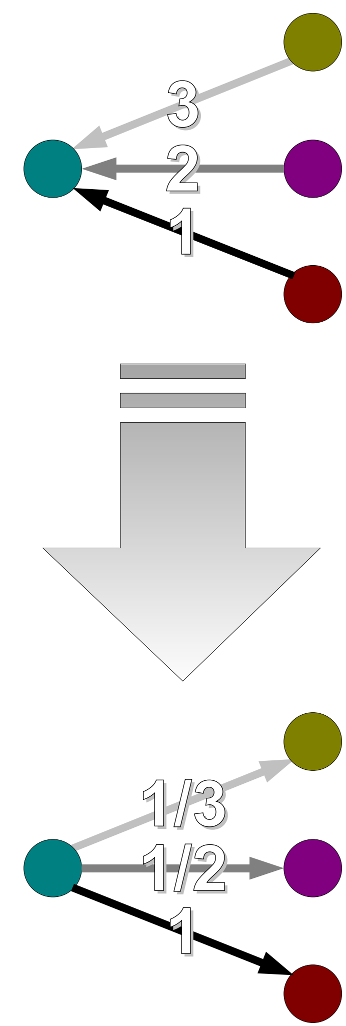
Persistence Effects

- **Inertia-like effect: past contacts may tend to become future contacts**
  - Unobserved relational heterogeneity
  - Availability to memory
  - (Compare w/autocorrelation terms in an AR process)
- **Simple implementation: fraction of previous contacts as predictor**
  - Log-rate of (i,j) contact adjusted by $\theta d_{ij}/d_i$



Recency/Ordering Effects

- **Ordering of past contact potentially affects future contact**
  - Reciprocity norms
  - Recency effects (salience)
- **Simple parameterization: dyadic contact ordering effect**
  - Previous incoming contacts ranked
    - Non-contacts treated as rank ∞
  - Log-rate of outgoing (i,j) contact adjusted by $\theta(1/\text{rank}_{ji})$



Triadic/Clustering Effects

- **Can also control for endogenous triadic mechanisms**
  - Two-path effects
    - Past outbound two-path flows lead to/inhibit direct contact (transitivity)
    - Past inbound two-path flows lead to/inhibit direct contact (cyclicity)
  - Shared partner effects
    - Past outbound shared partners lead to/inhibit direct contact (common reference)
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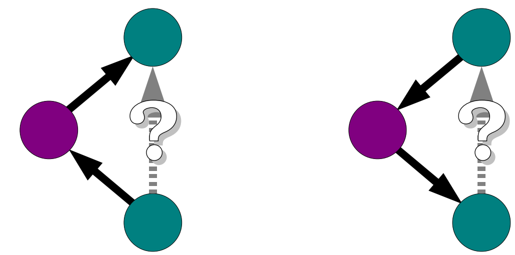
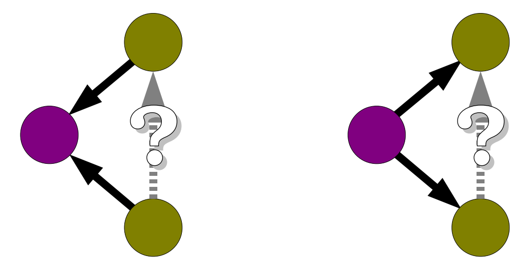


Two-Path Effects



Shared Partner Effects

Coordination, Hub Status and Institutional Role

- **Importance of coordination well-known among practitioners (e.g., Auf der Heide, 1989)**
  - Response organizations include institutionalized coordinative roles, e.g., dispatchers, call desk operators
- **Can hub status be explained via existing roles?**
  - “Institutionalized” vs. “emergent” coordinators (*a la* Dynes, 1970)
- **Coding from transcript content**
  - Title includes “command,” “desk,” “operator,” “dispatch,” “manager,” “control,” or “base”
  - Responder identified with site (“Newark Airport”)