Nested Case Control Sampling for Egocentric and Relational Cox Models

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1. Motivation

- Partial likelihoods for egocentric and relational Cox models:

$$PL(\beta) = \prod_{e=1}^{E} \left\{ \frac{exp[\beta' s(i_e, t_e)]}{\sum_{j \in R(t_e)} exp[\beta' s(j, t_e)]} \right\}$$

  - E is the number of events during the observation time.
  - $R(t_e)$ is the set of nodes or edges at risk at time t_e.
  - $s(j, t_e)$ is the covariate vector of node or edge j.
  - Running times of parameter β estimation algorithms are $O(EN)$ and $O(EN^2)$, respectively where N is the number of nodes.

- Factors N and N^2 can be reduced significantly if network covariates under consideration allow for sparse updates of sum denominators.

- For larger networks and richer sets of covariates, we need to consider other approaches including sampling-based and online inference methods.

2. The Risk Set Sampling Framework

- We denote the network history up to, but not including, time t as H_{t^-}.

- At time t_e when an event occurs on node or edge i_e (case), based on information in H_{t^-} we will sample a subset $\tilde{R}(t_e)$ (controls) from the current risk set $R(t_e)$. The case is always included in the sampled risk set.

- The modified partial likelihoods [Borgan et al, 1995]:

$$PL_s(\beta) = \prod_{e=1}^{E} \left\{ \frac{exp[\beta' s(i_e, t_e)] w_{i_e}(t_e)}{\sum_{j \in \tilde{R}(t_e)} exp[\beta' s(j, t_e)] w_j(t_e)} \right\}$$

  - Each sampled individual is weighted by $w_j(t_e)$ to compensate for differences in sampling probabilities.
  - Variant sampling designs based on H_{t^-} will results in different definitions of $w_j(t_e)$.
  - Packages for weighted conditional logistic regression models can be used for parameters estimation.

- In the $1{:}m$ nested case-control sampling design, each sampled risk set will contain the case and $m-1$ controls which are randomly sampled (without replacement) from the current risk set, i.e. $w_j(t_e)$ are equal for all j.

3. Egocentric Twitter-Vaccine Data

- We consider a network of 101,853 Twitter users from 12/5/2009 to 1/19/2010 [Salathé et al, 2011].

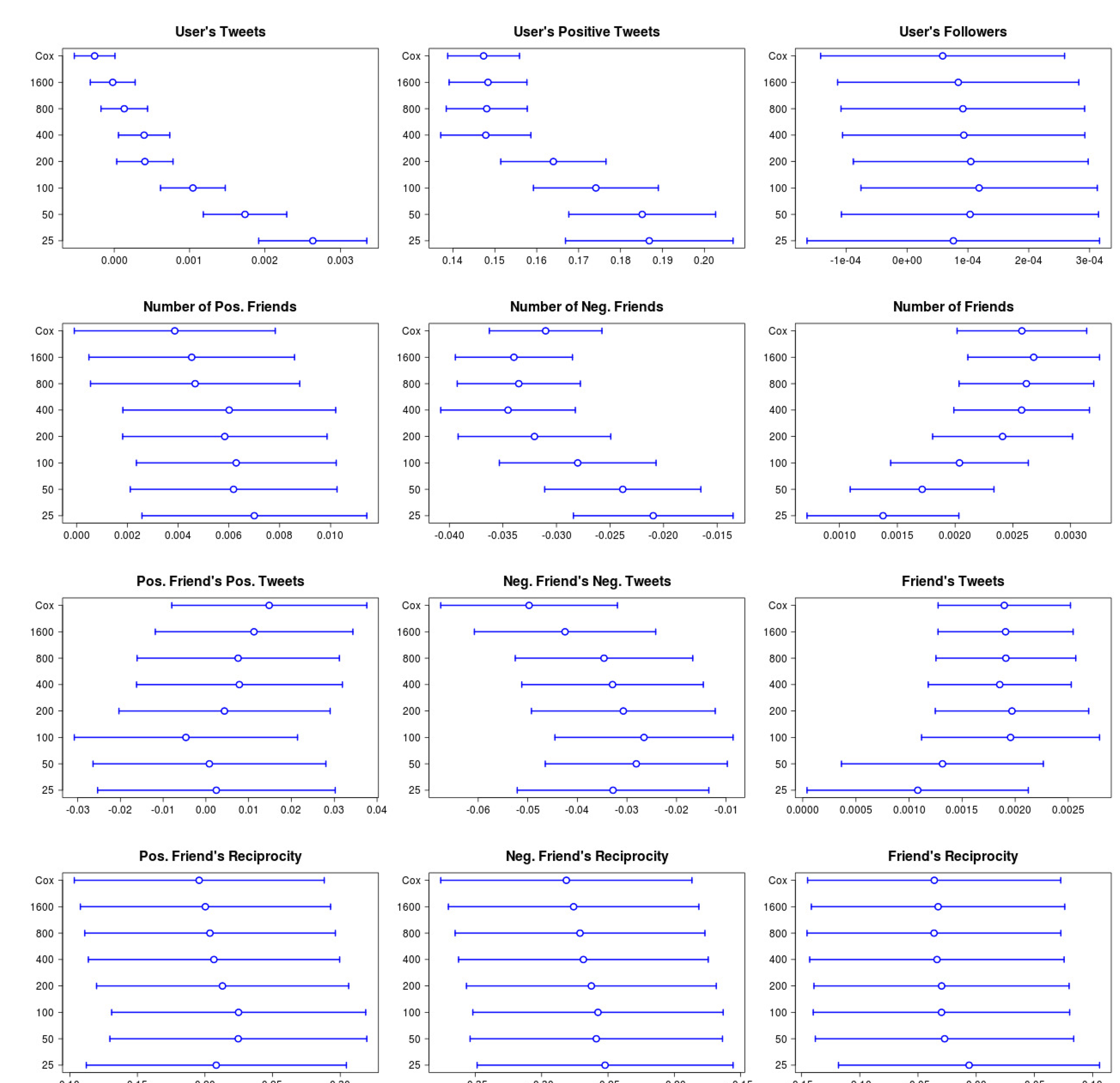
- There are 4,619,852 following edges among these users who have made 53,300 tweets about influenza A(H1N1) vaccine.

- Each tweet is classified as + (6,416), - (3,510), or neutral (43,374).

- We are interested in how the *positive* sentiment of future tweets of a user is associated with her past tweeting behavior as well as her friends' ones.

- Some representative network covariates:
  - User's past tweeting behavior: the current numbers of total and + tweets.
  - Friends' past tweeting behaviors:
    - ➢ The current numbers of + and - friends (weak).
    - ➢ The current numbers of + and - reciprocated friends (strong).
    - ➢ The current numbers of + and - tweets that these friends have made.

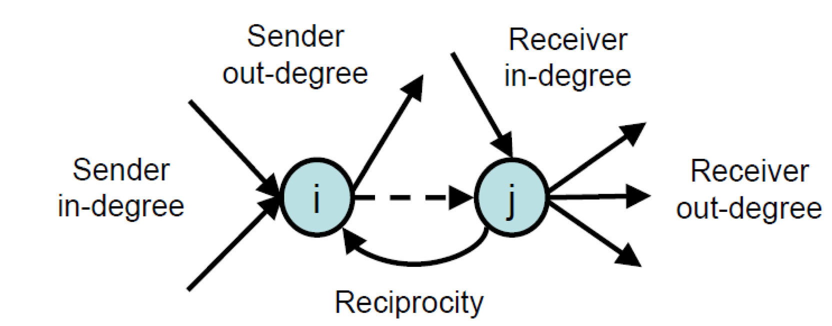
- 95% confidence intervals of coefficient estimates with different sizes of nested controls m:

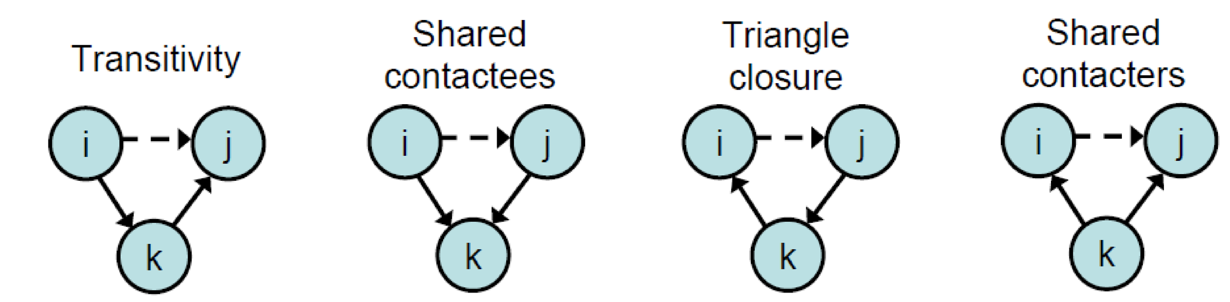


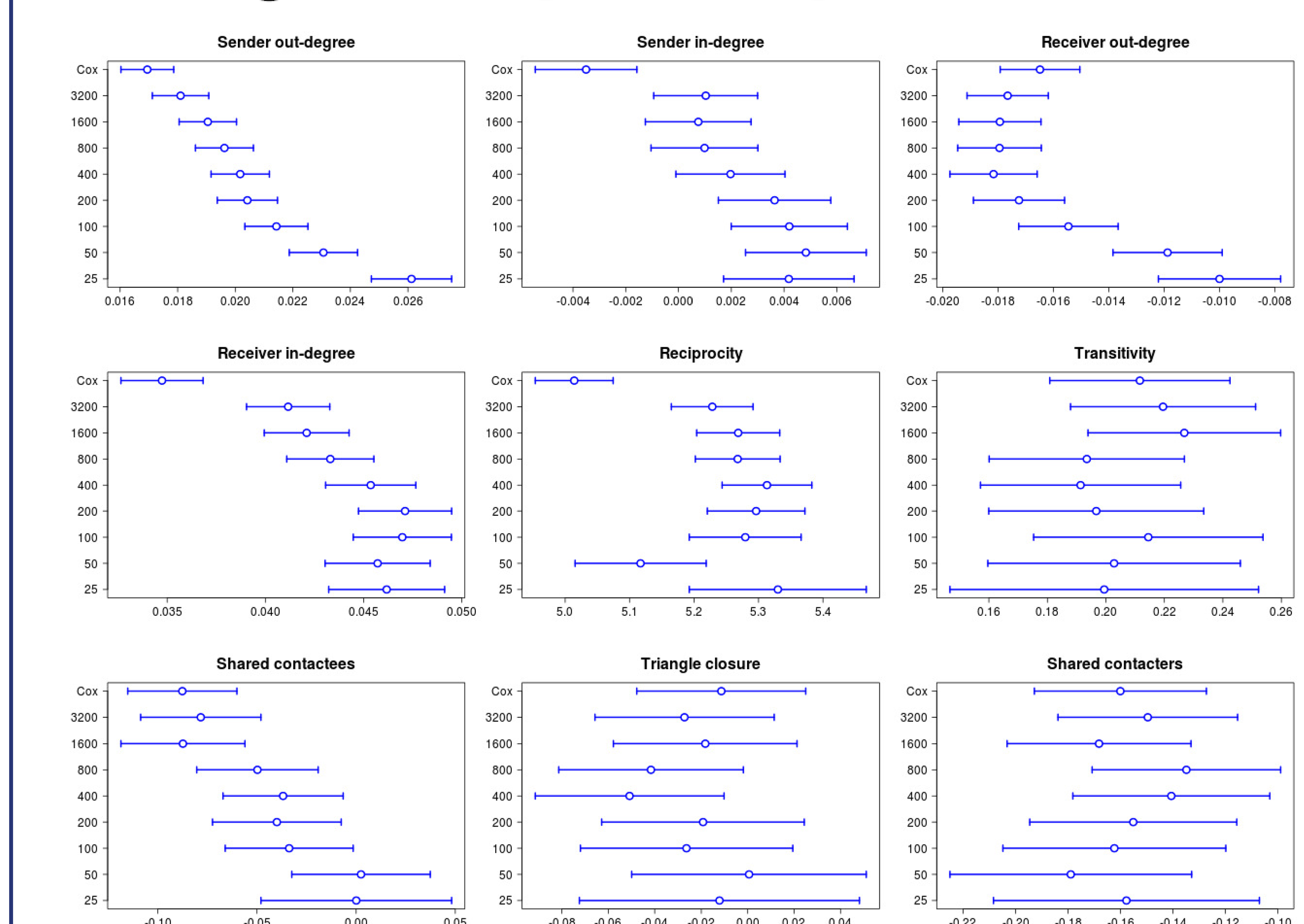
4. Relational Irvine-Facebook Data

- We consider the network formation process of an online social network at UC Irvine [Opsahl et al, 2009] from 5/12/04 to 6/1/04 (7,645 edge formation events among 1,596 active users, i.e. ≈ 2.5 millions of potential edges).

- Network covariates:







5. Conclusions and Future Work

- Nested case-control sampling is simple to implement and fast though biased estimates are possible.

- Other more adaptive risk set sampling methods such as counter-matching will be explored to reduce estimation biases.

- The performance of these sampling methods in the prediction task will also be considered.

- The framework can be also applied to Aalen models with time-varying coefficients [Zhang et al, 1999].