New Directions in Greedy Routing on Social Networks

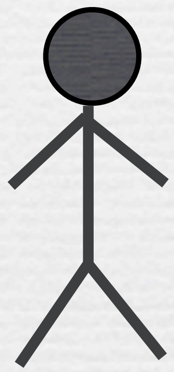
The Membership Dimension

David Eppstein, Michael Goodrich, Maarten Löffler,
Darren Strash, and presented by Lowell Trott

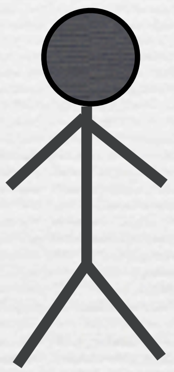
The Background

- Milgram's Small World experiment
  - 6 degree's of separation, or Kevin Bacon
  - Omaha or Wichita to Boston

The Experiment

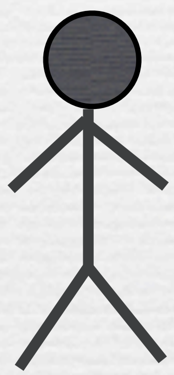


You



Someone in Boston

The Experiment



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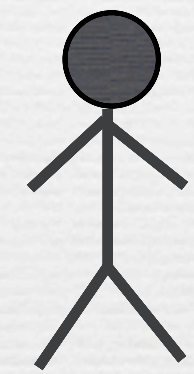
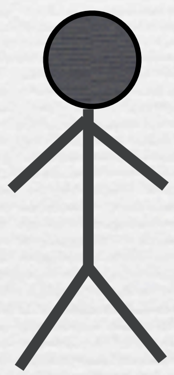


Someone in Boston

Name
Occupation
etc.

The Experiment



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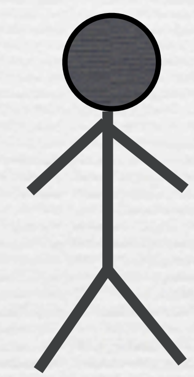
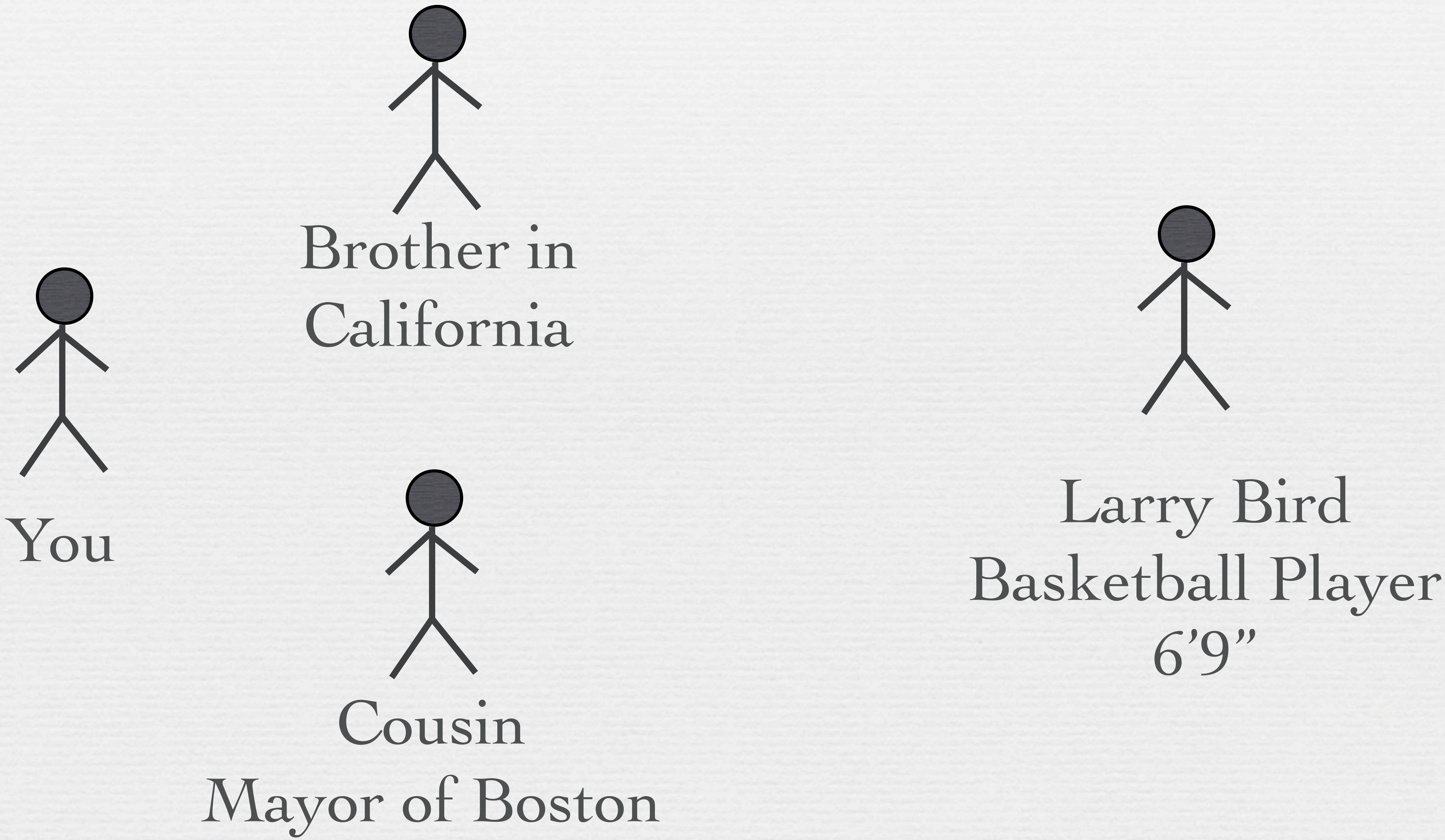
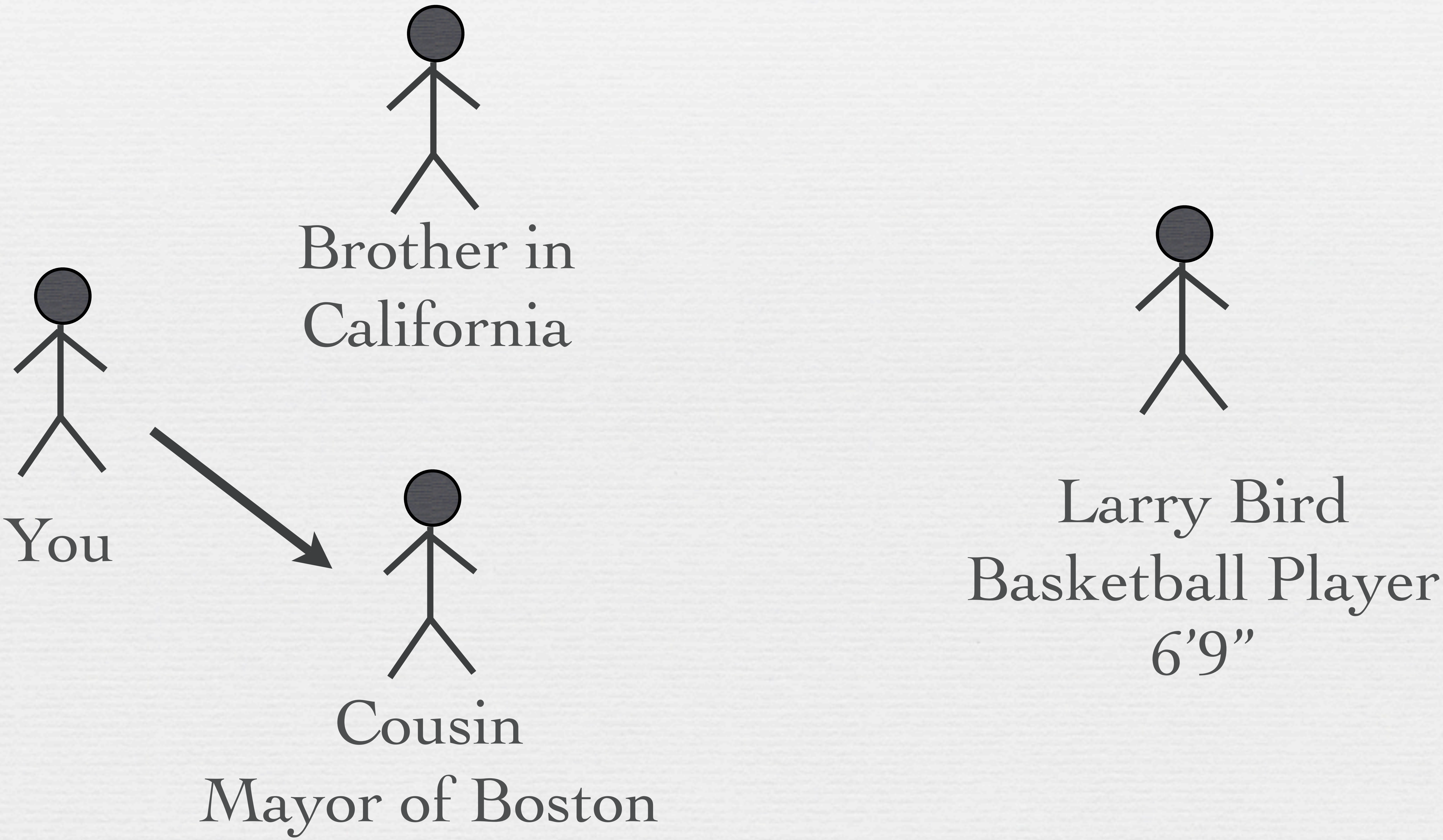


Larry Bird
Basketball Player
6’9”

The Experiment



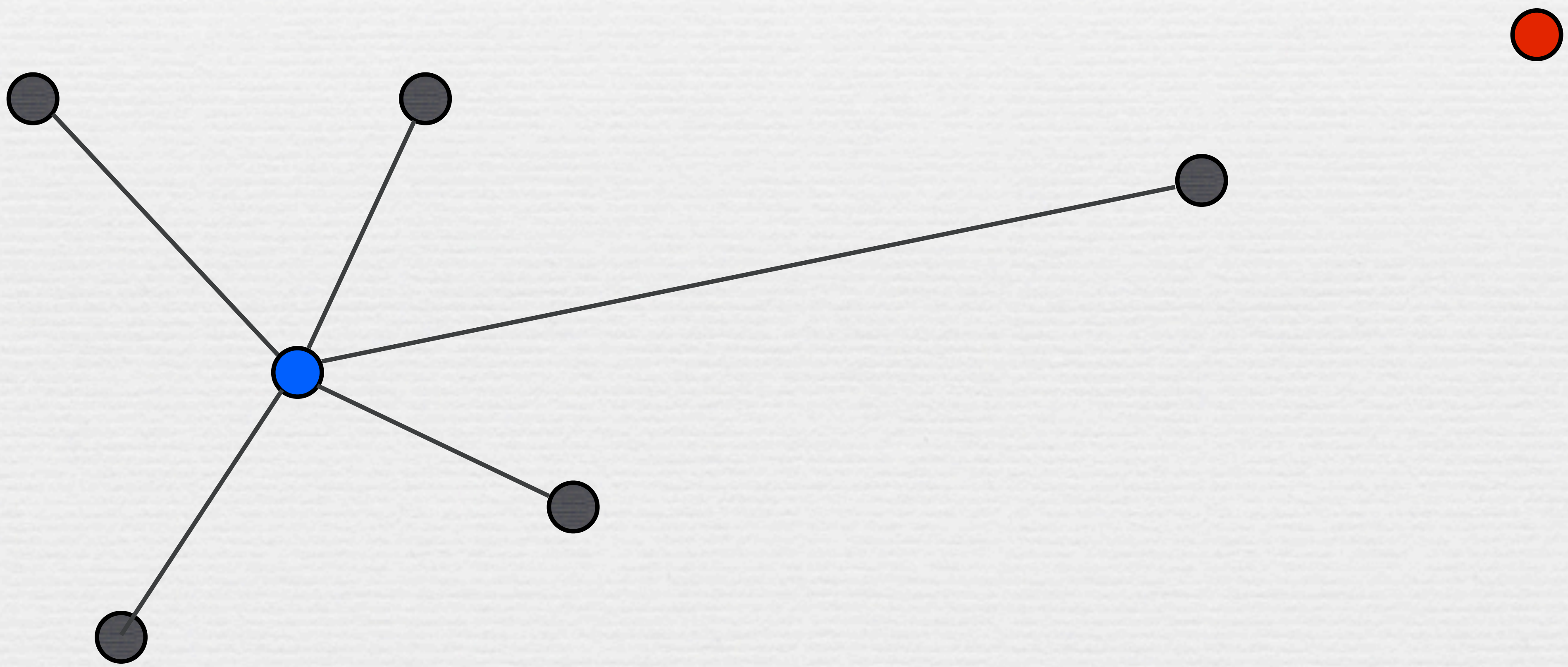
The Experiment



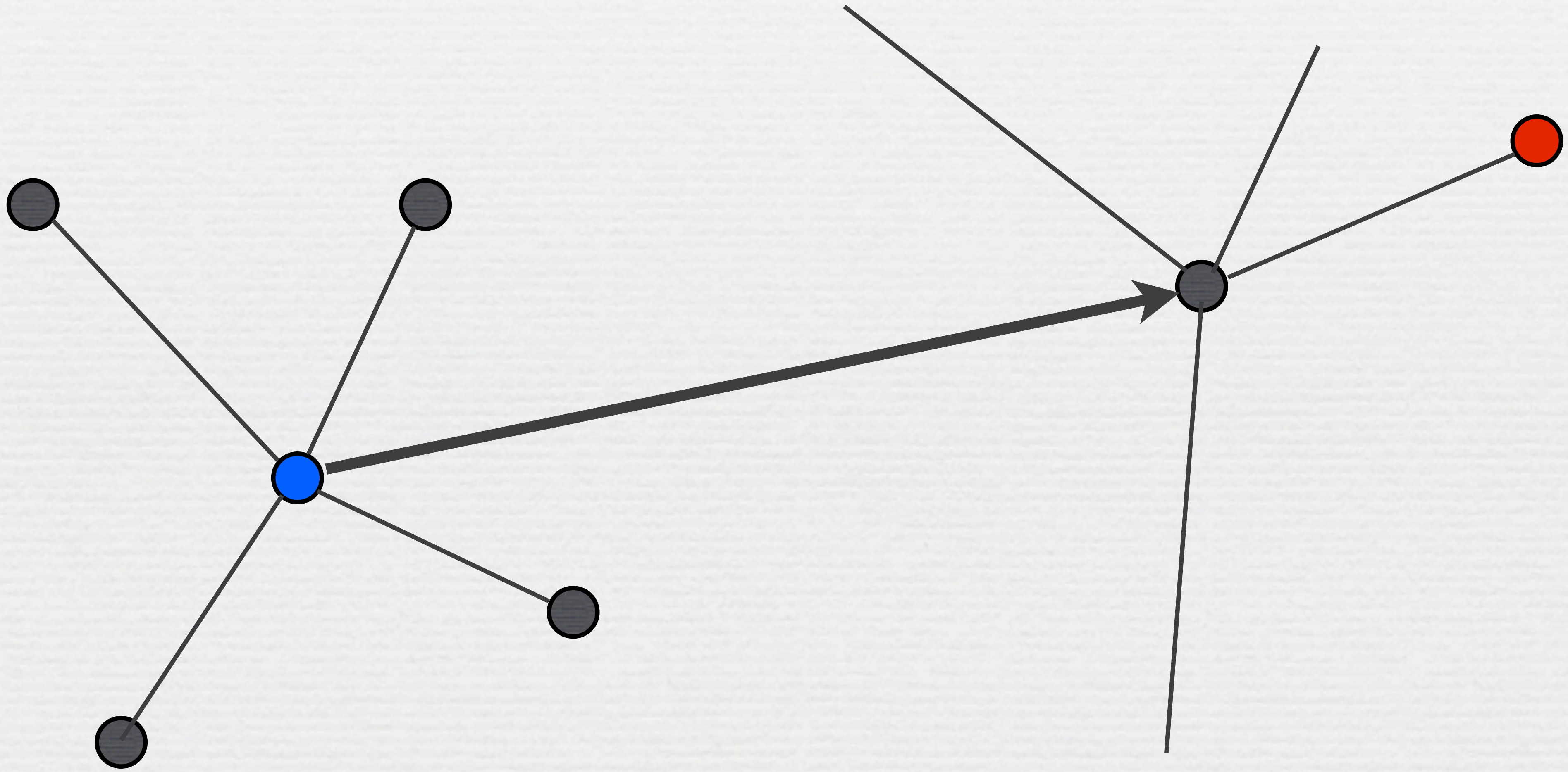
Greedy Routing

- Only local knowledge of the network
- To reach the solution
    - Move to the neighboring node that is "closer" to the target node

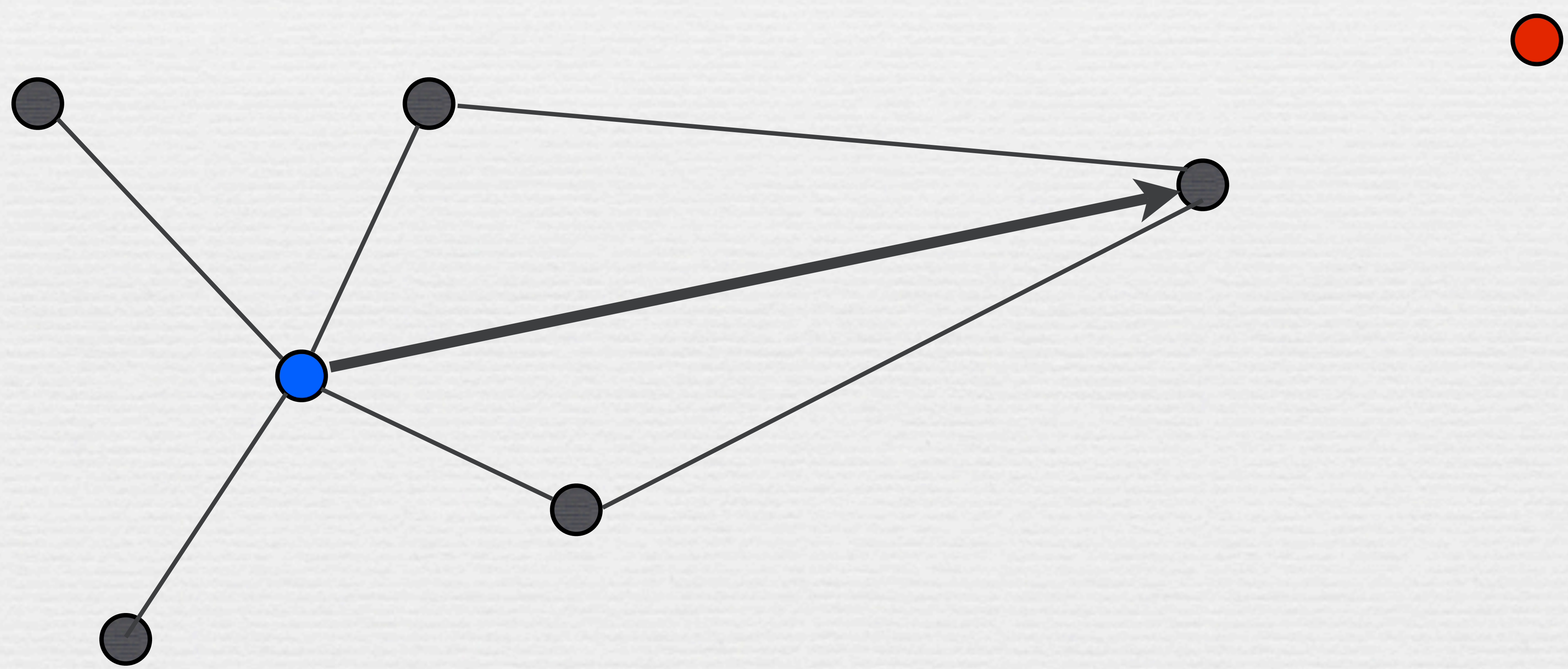
Greedy Routing



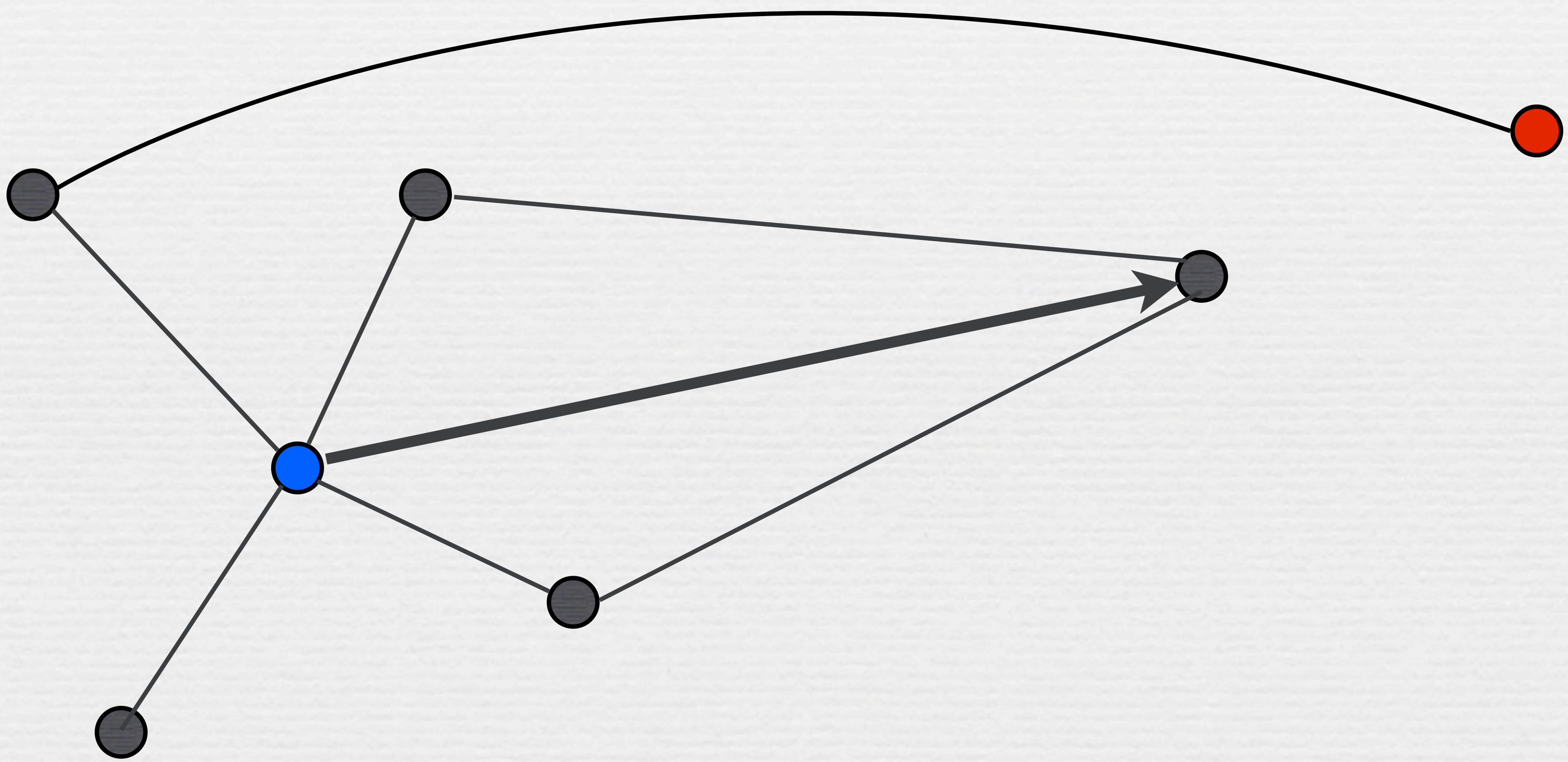
Greedy Routing



Greedy Routing



Greedy Routing



The Problem Is Not...

- Creating a Greedy Embedding
  - Eppstein and Goodrich can succinctly embed in the hyperbolic plane
  - Goodrich and Strash can succinctly embed 3-connected planar graphs in the Euclidean plane

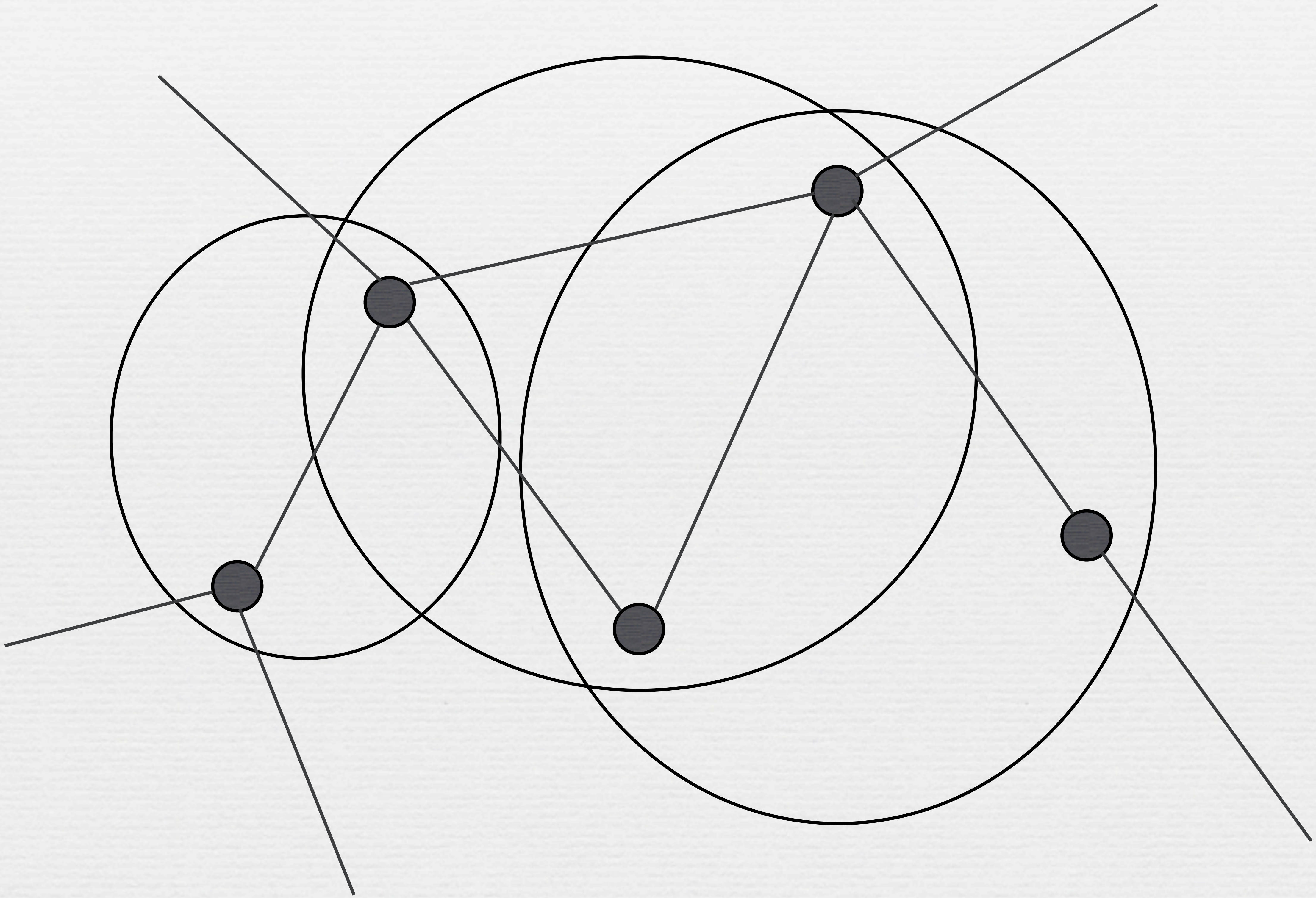
The Problem Is...

- Creating a Greedy Embedding that reflects the method used by people to route in their own social networks
    - Excludes the use of sophisticated techniques
    - Maintain simple complexity, especially at a local level

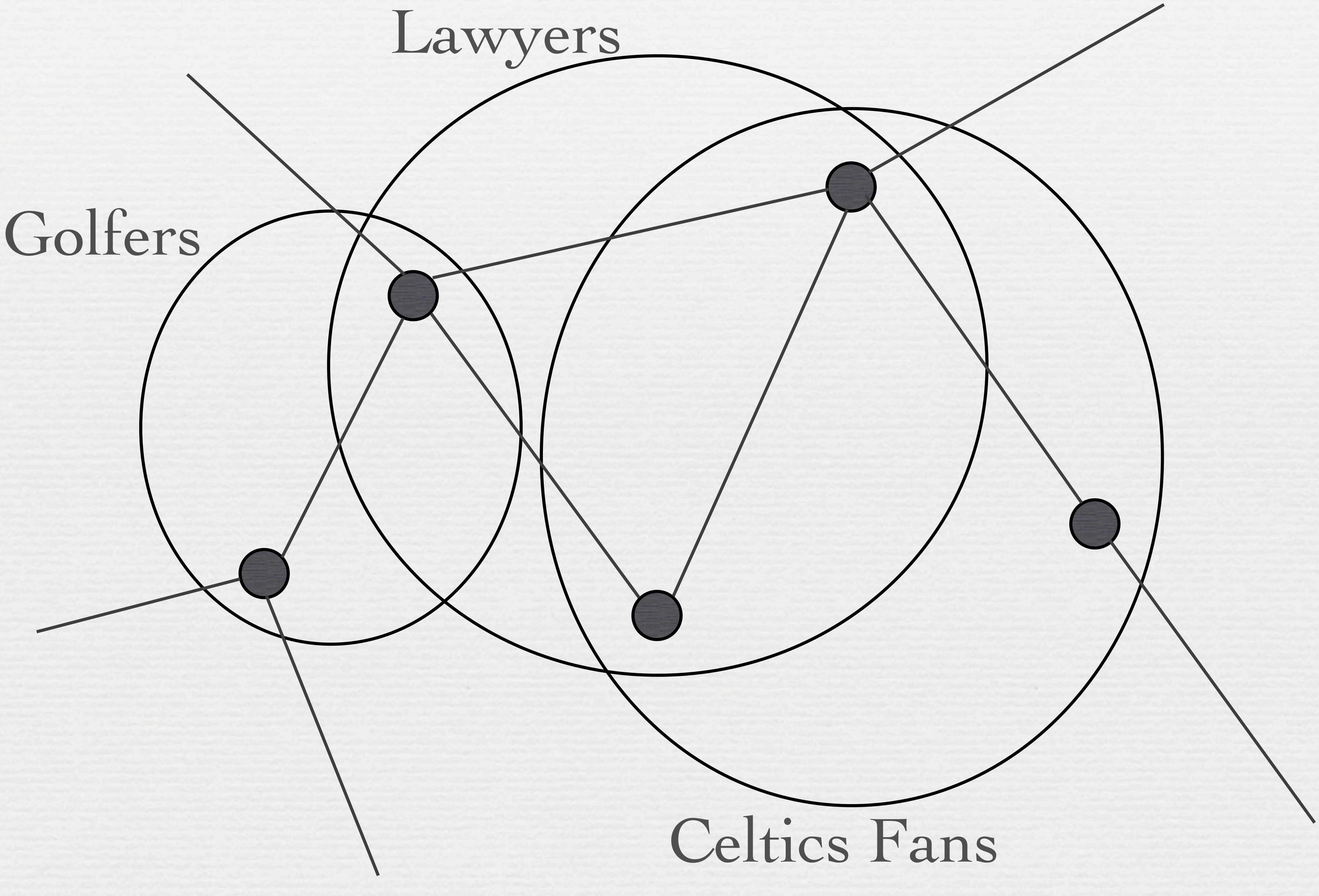
What We Want

- Given a network, we want an embedding that always allows our network to perform greedy routing
  - 232 of the 296 letters started by Milgram never reached their destination
- We consider a method that
  - is based on categorical group membership
  - Then we want each node to belong to few sets

Group Membership



Group Membership



Grouping Concepts

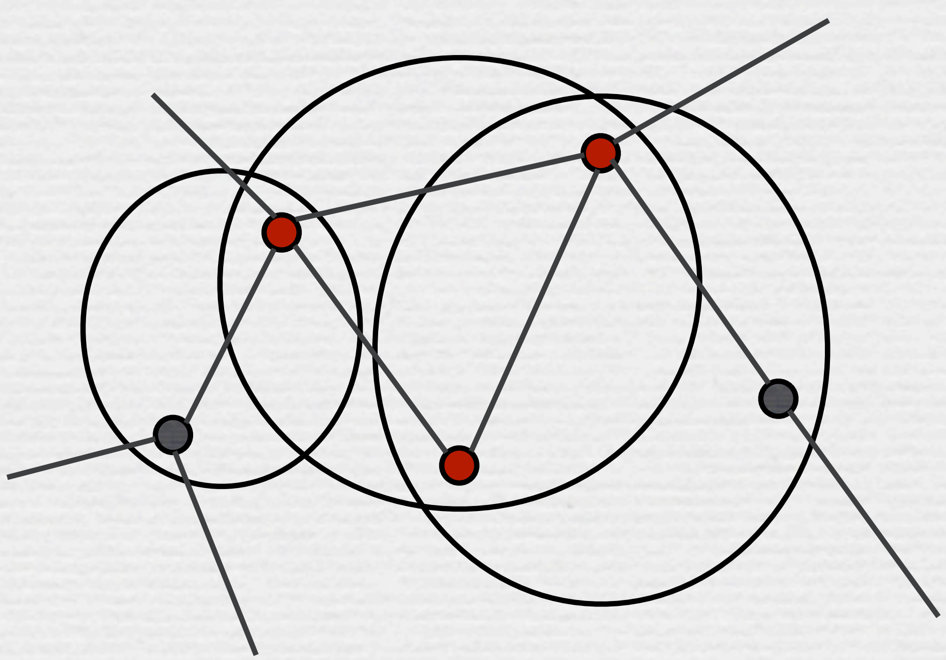
- Internally Connected
  - For a graph and set of groups, each subgraph within a group is connected
- Shattered
  - For each vertex, it has a neighbor that belongs to a group with the target that it does not belong to

Grouping Concepts

- Internally Connected
  - Seems natural, but may not be necessary
- Shattered
  - Necessary
- Both guarantee greedy routing will always work

Membership Dimension

- The Membership Dimension of our groupings is the maximum number of groups any node in our network belongs to



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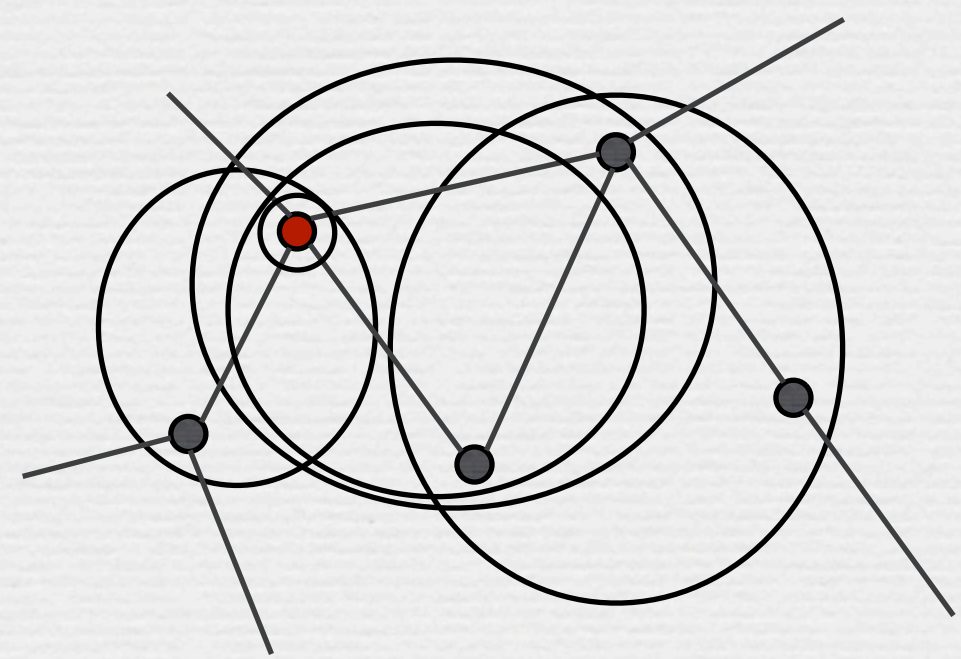

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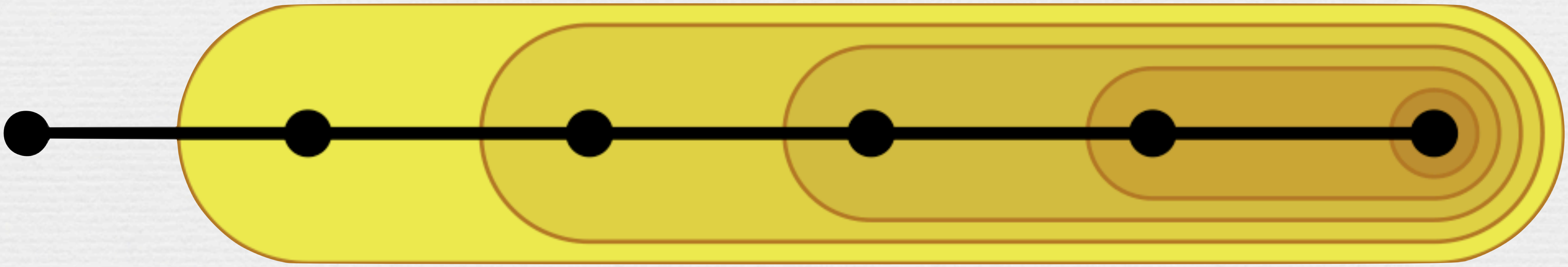
Desired Construction

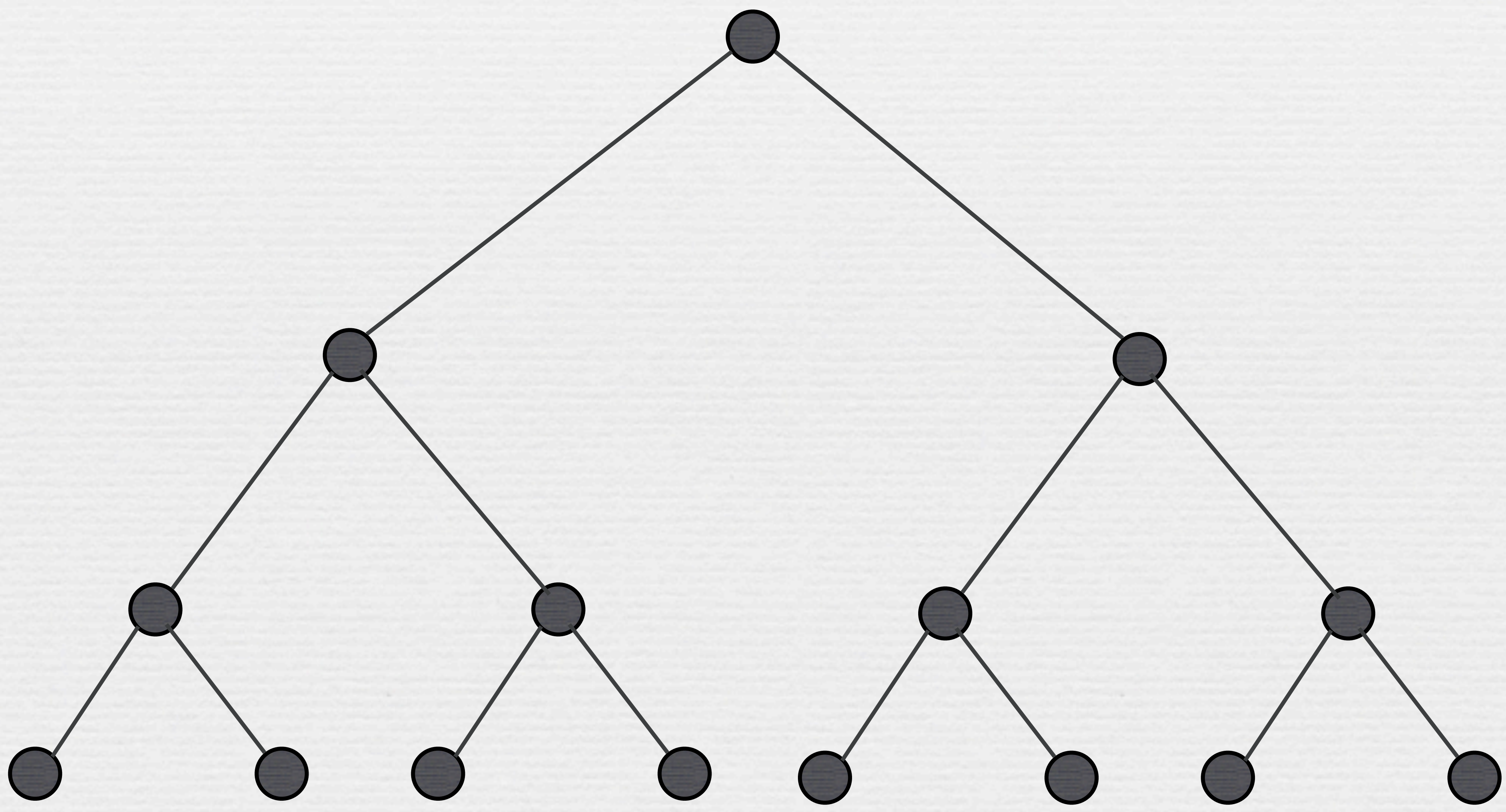
- We are trying to build S, our set of groups, for a graph G such that
  - (G,S) is shattered
  - (G,S) is internally connected
  - The membership dimension of S is minimized

Group Construction for Paths

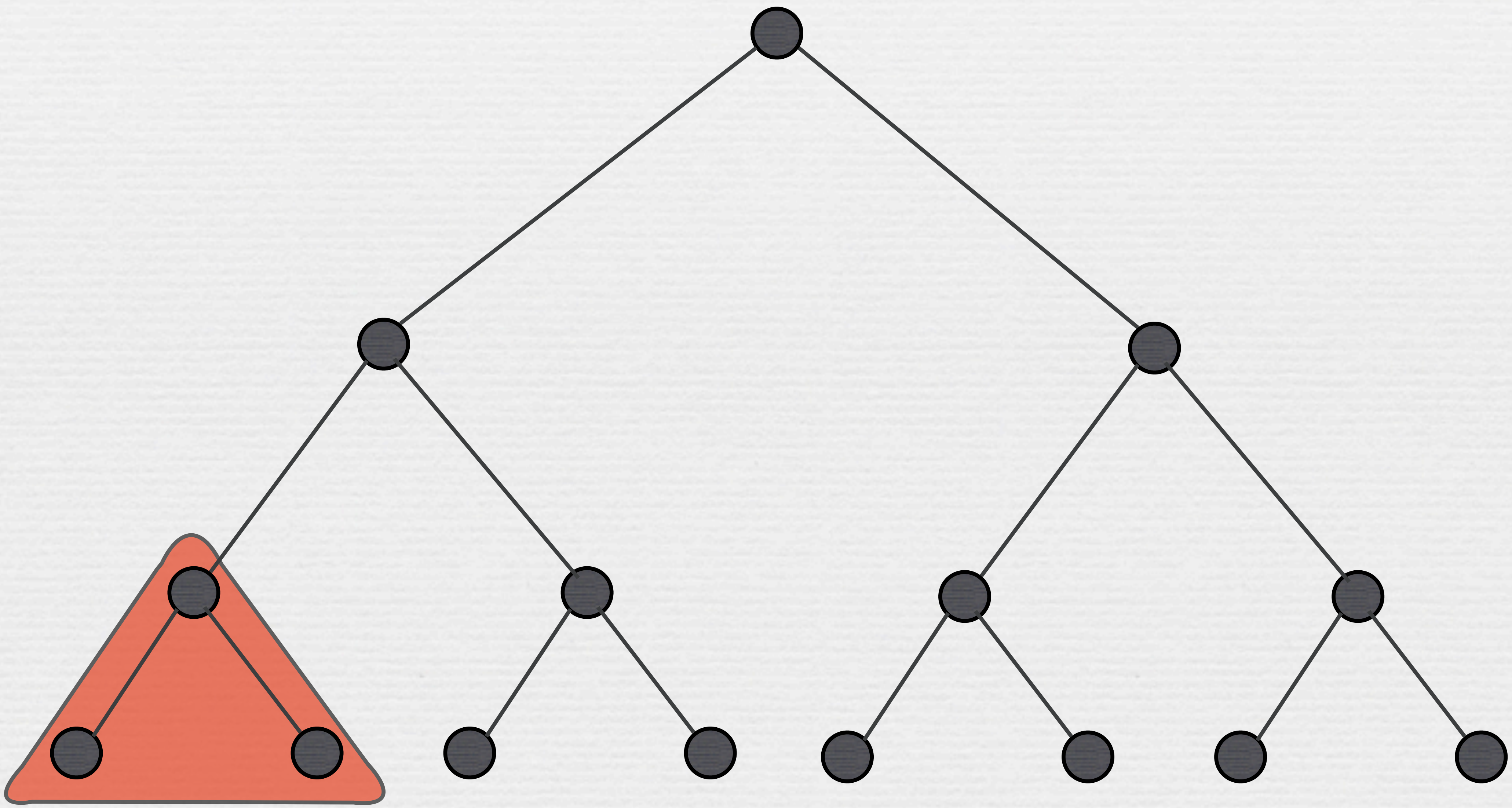


Thus a graph G will have minimum membership dimension of the diameter of G

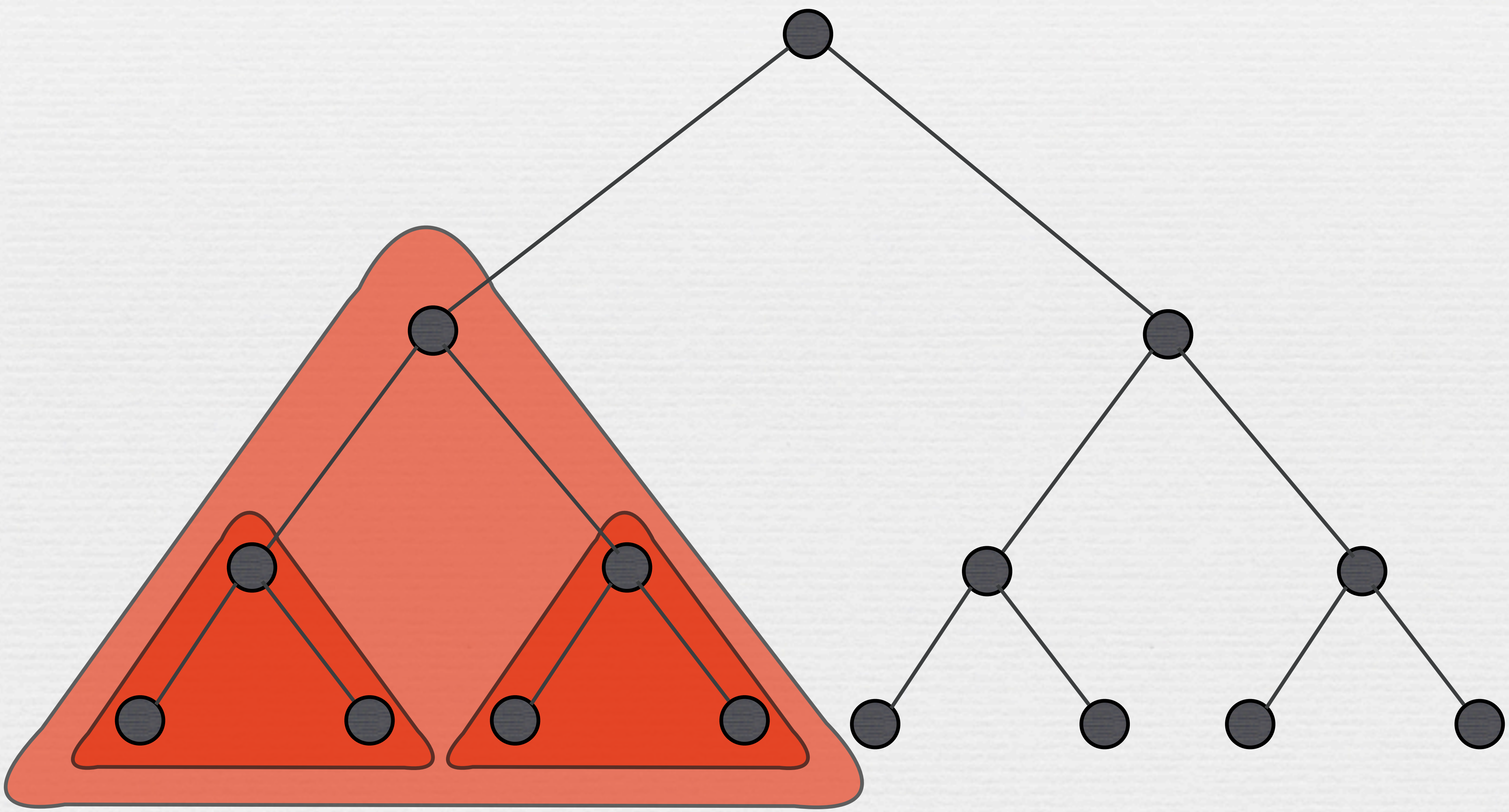
Tree Construction



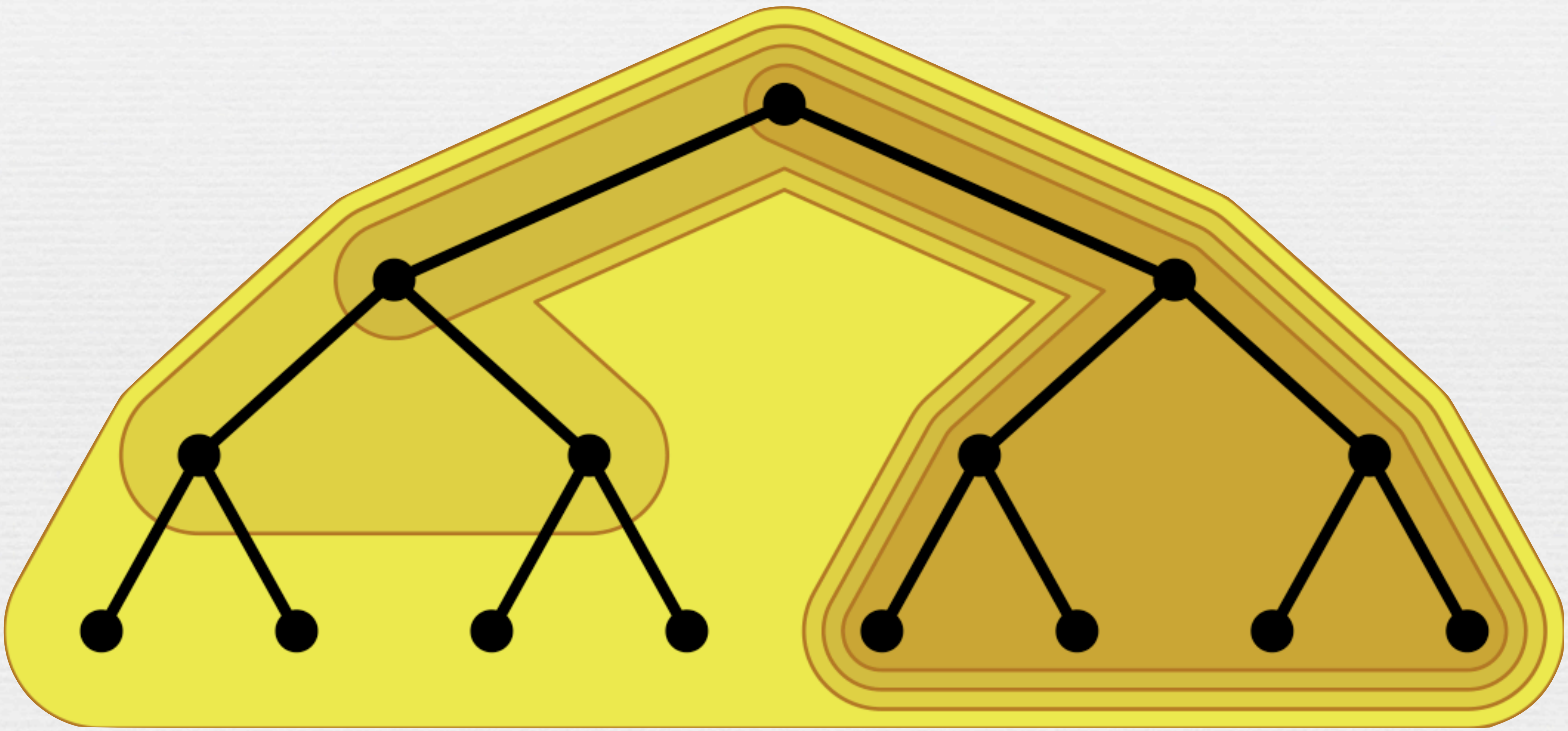
Routing Down



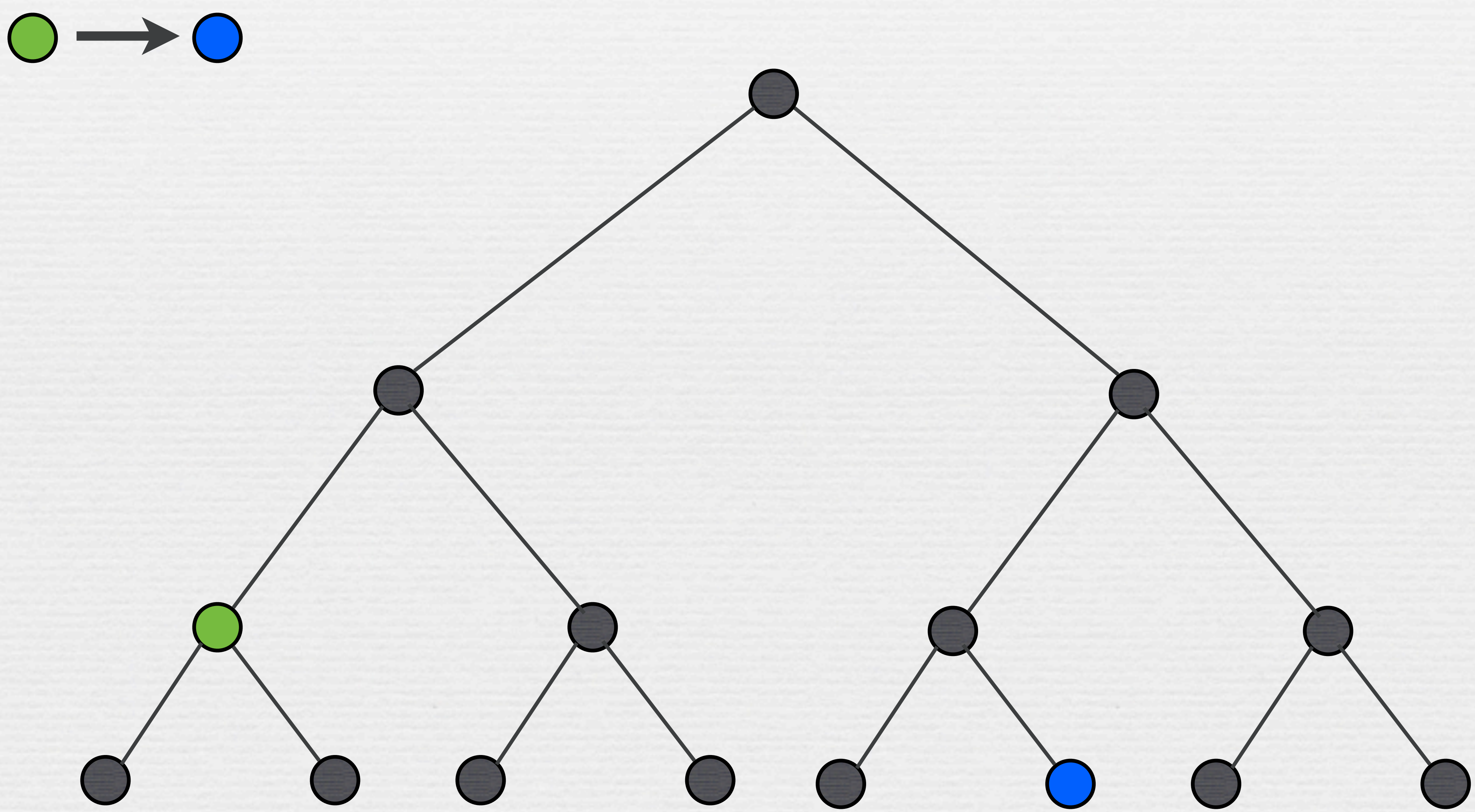
Routing Down



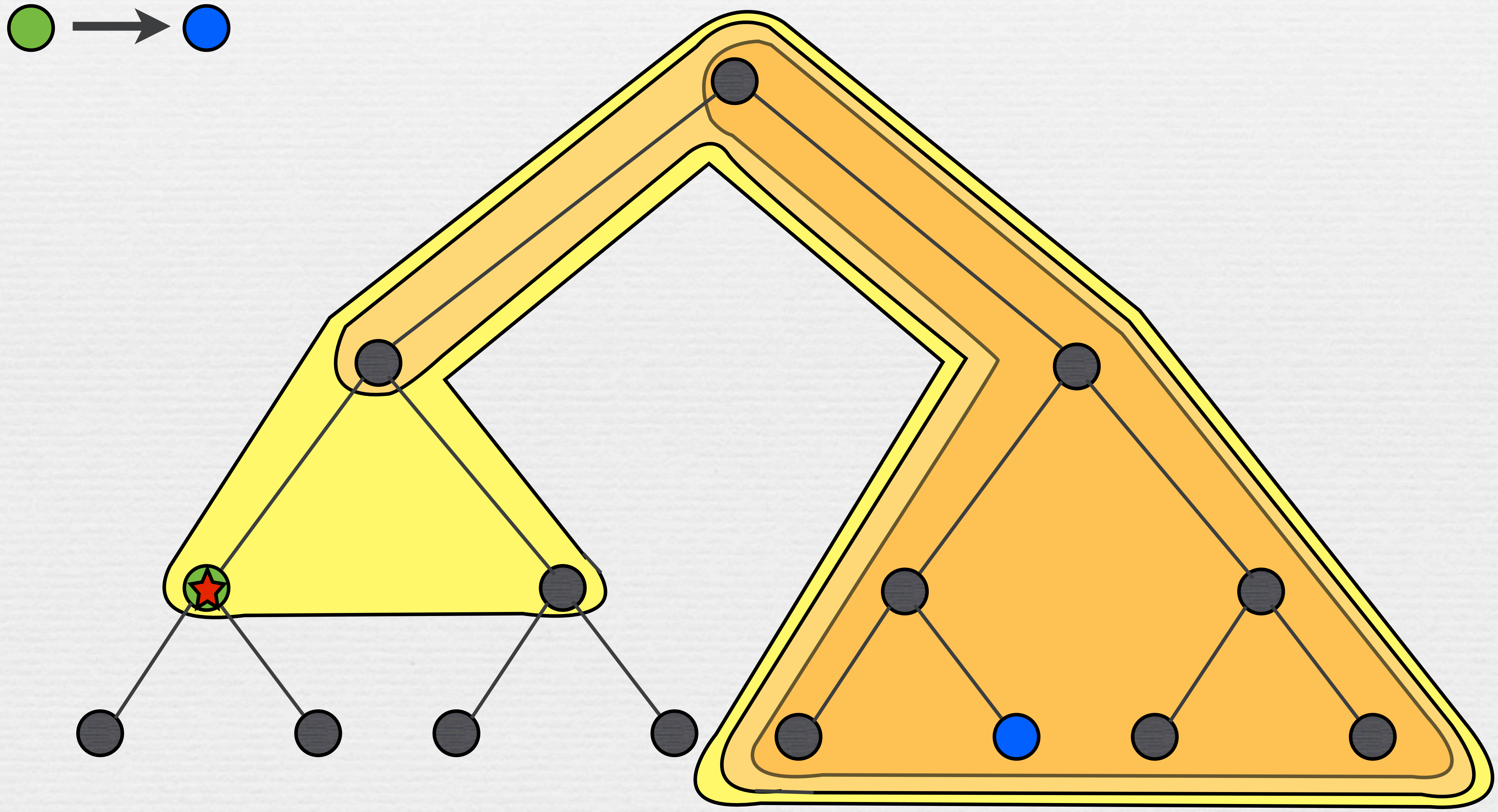
Routing Up



Example



Example



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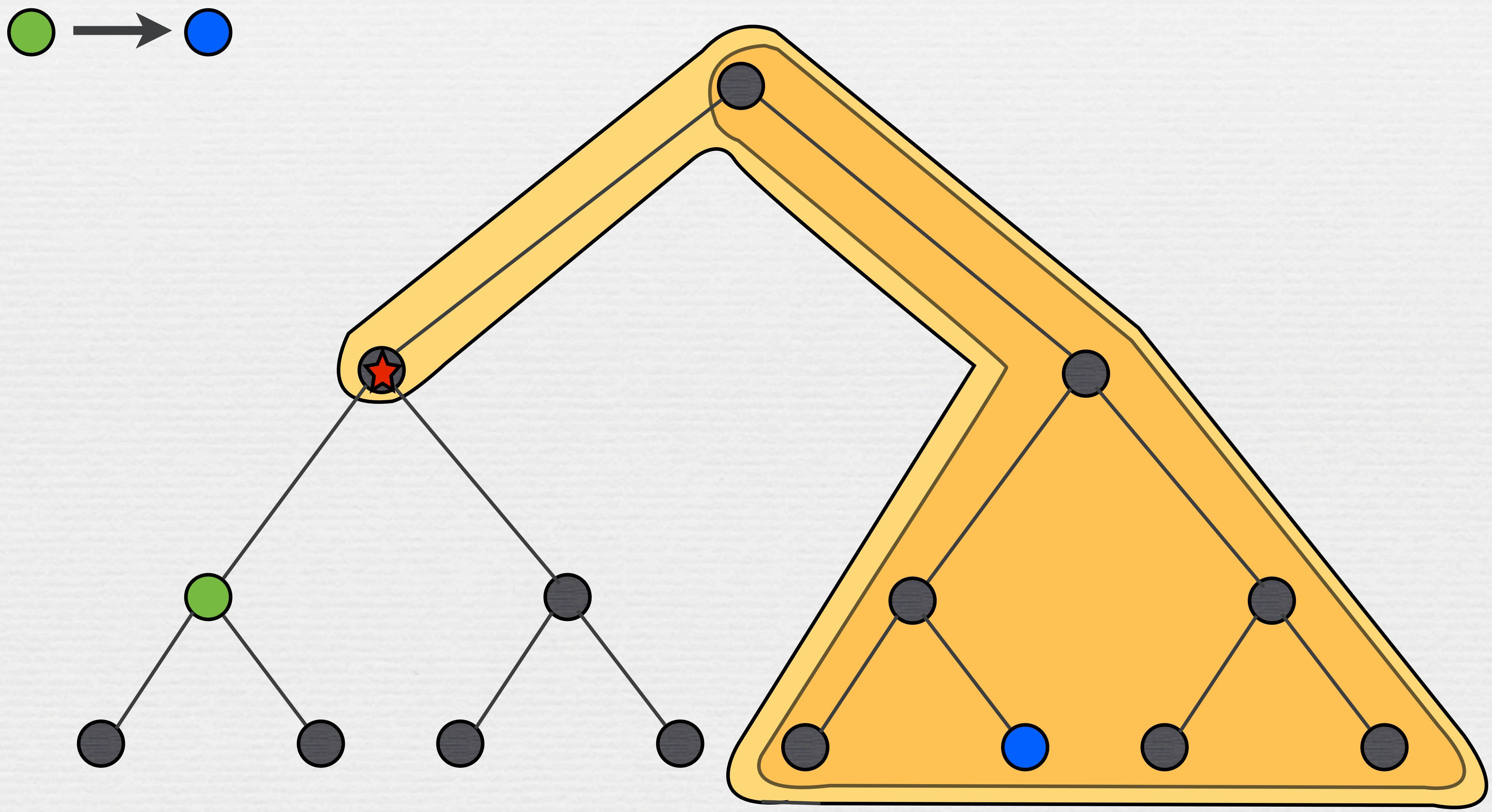

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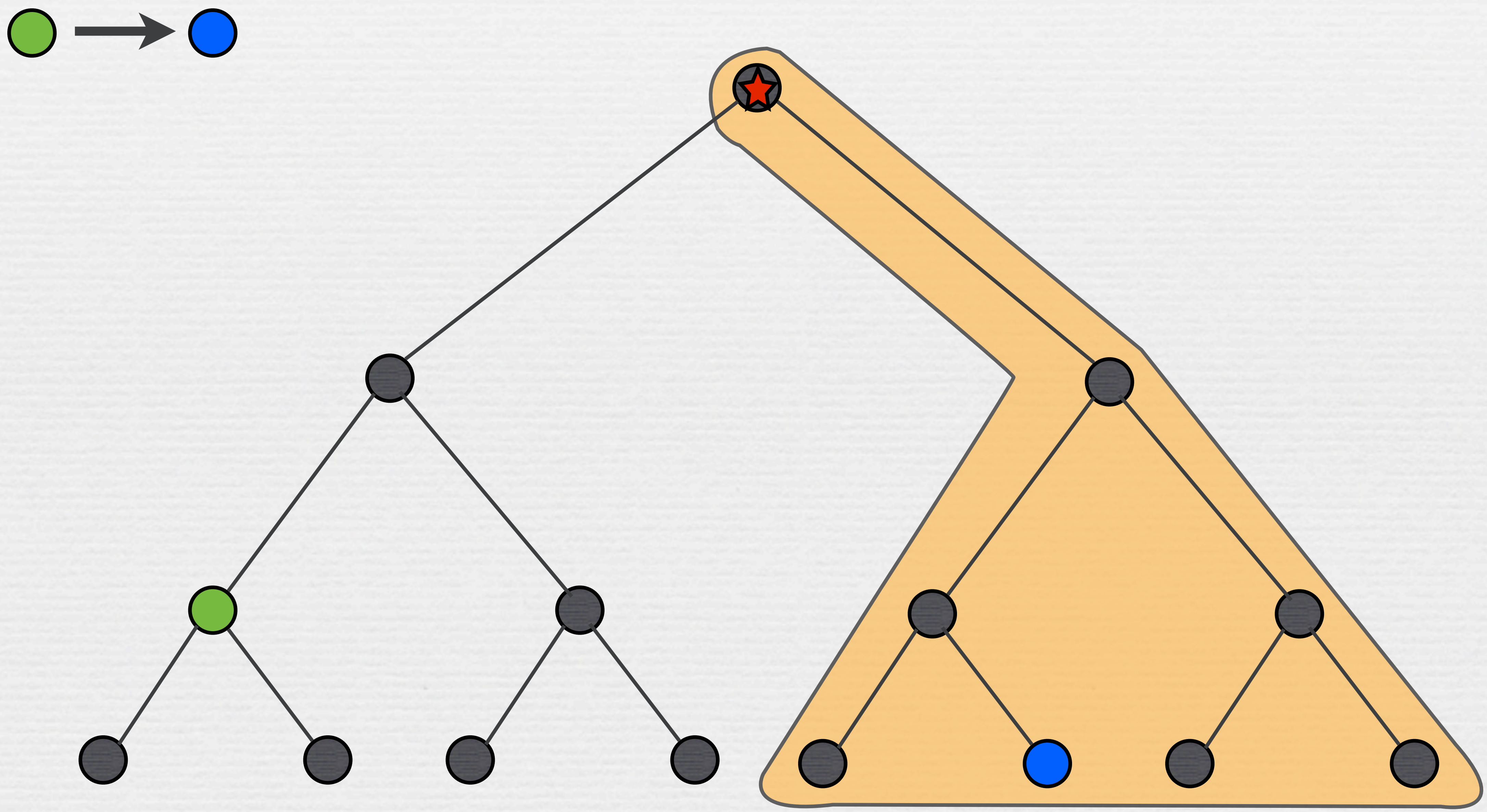
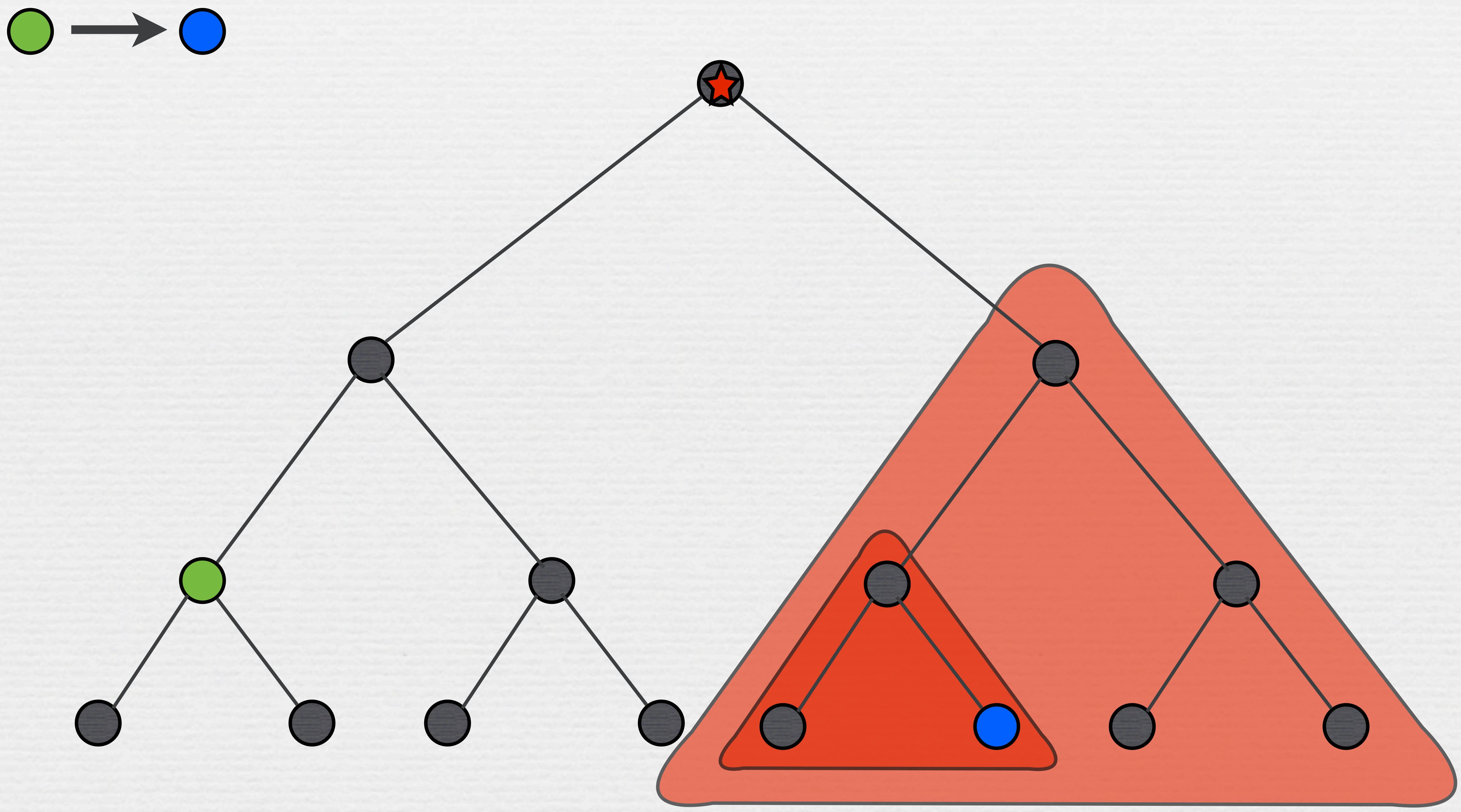
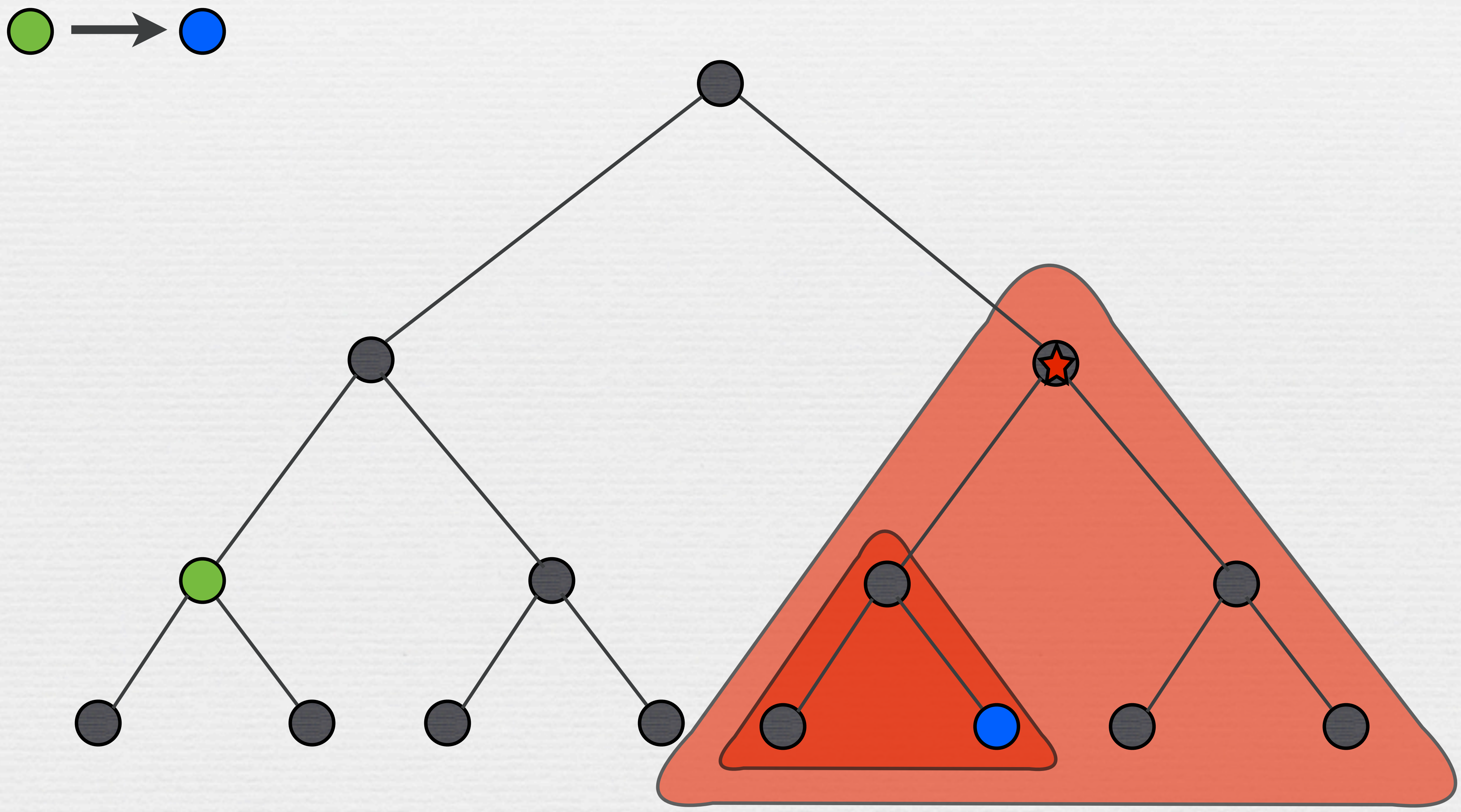


Example



Example



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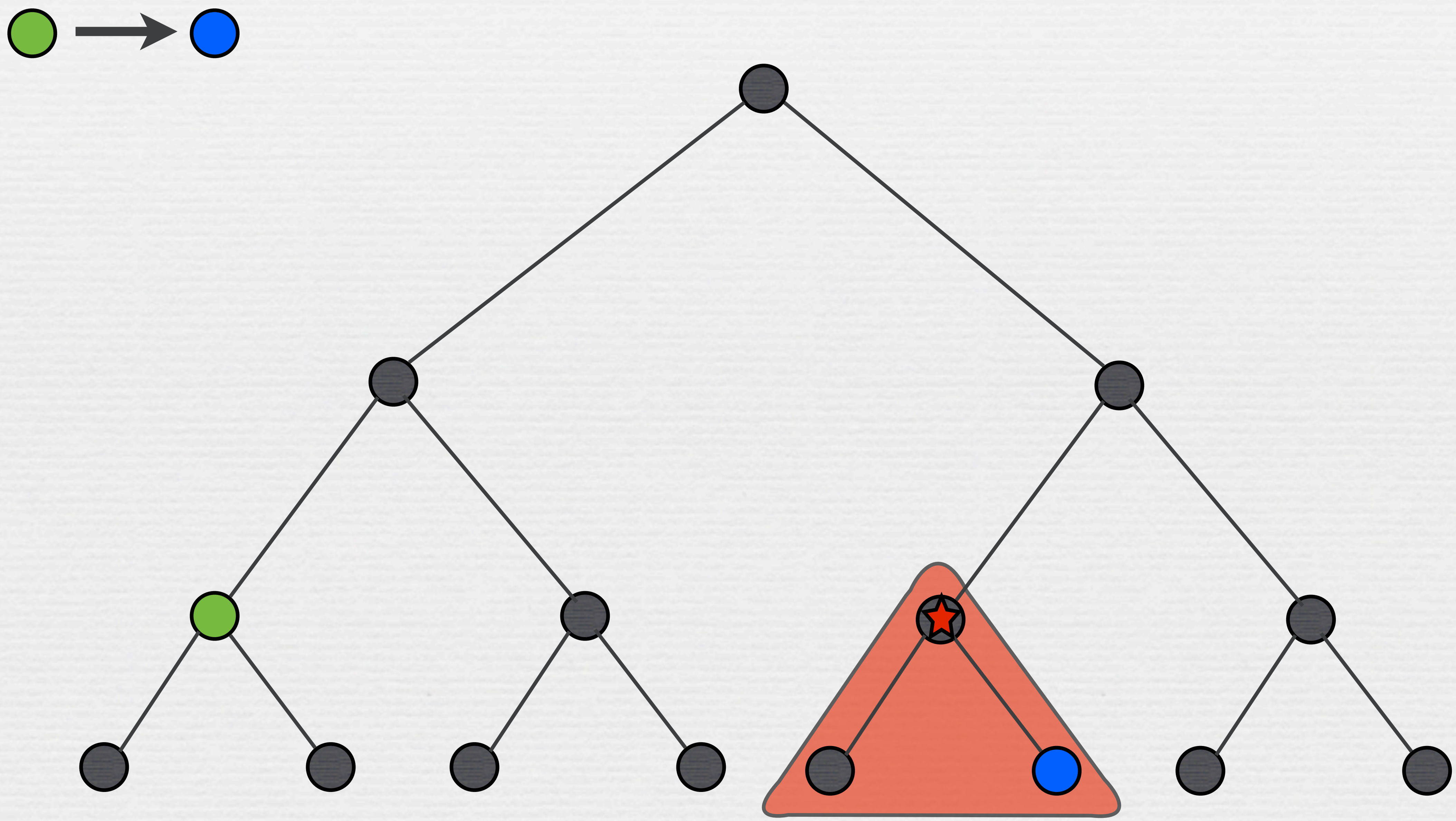

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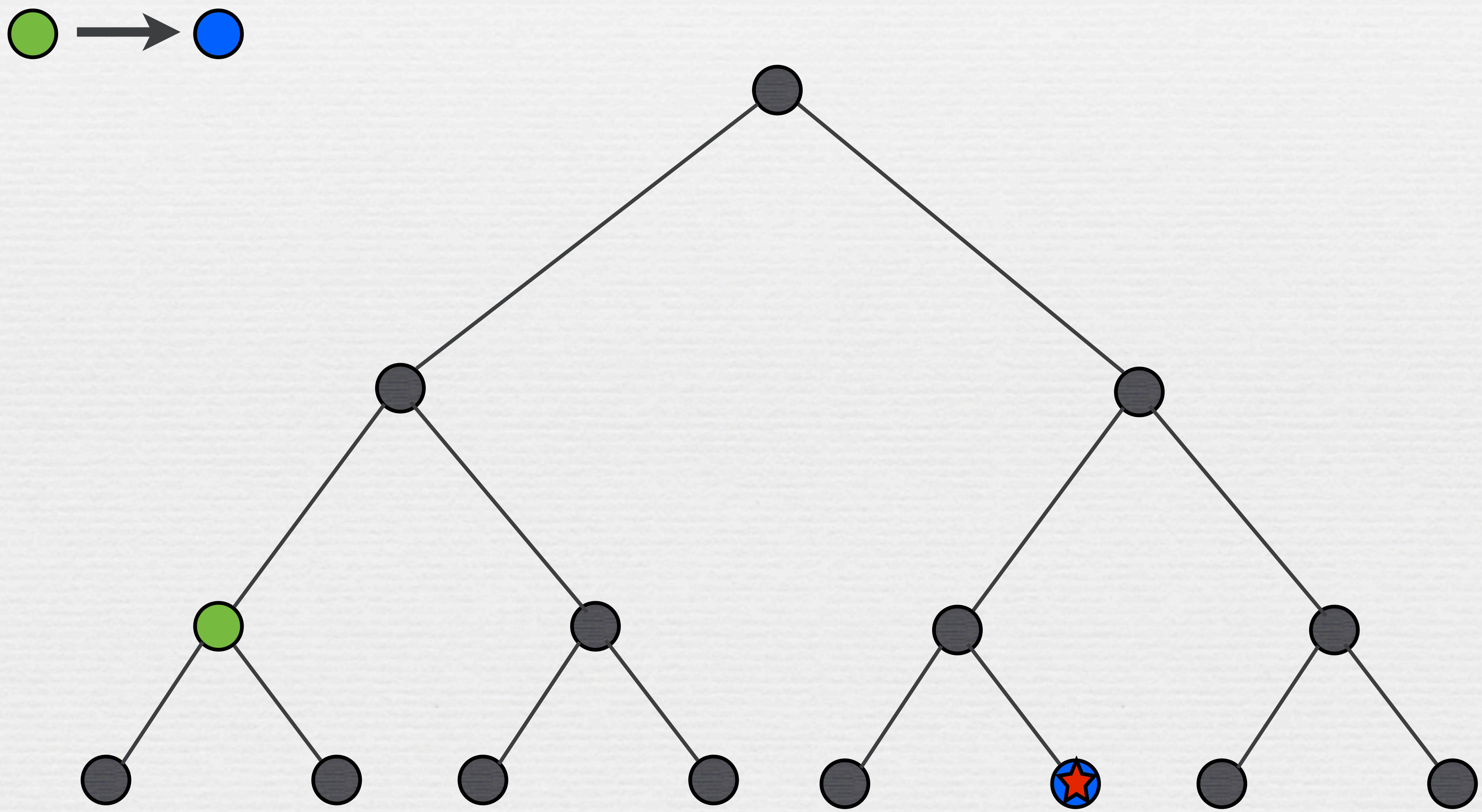


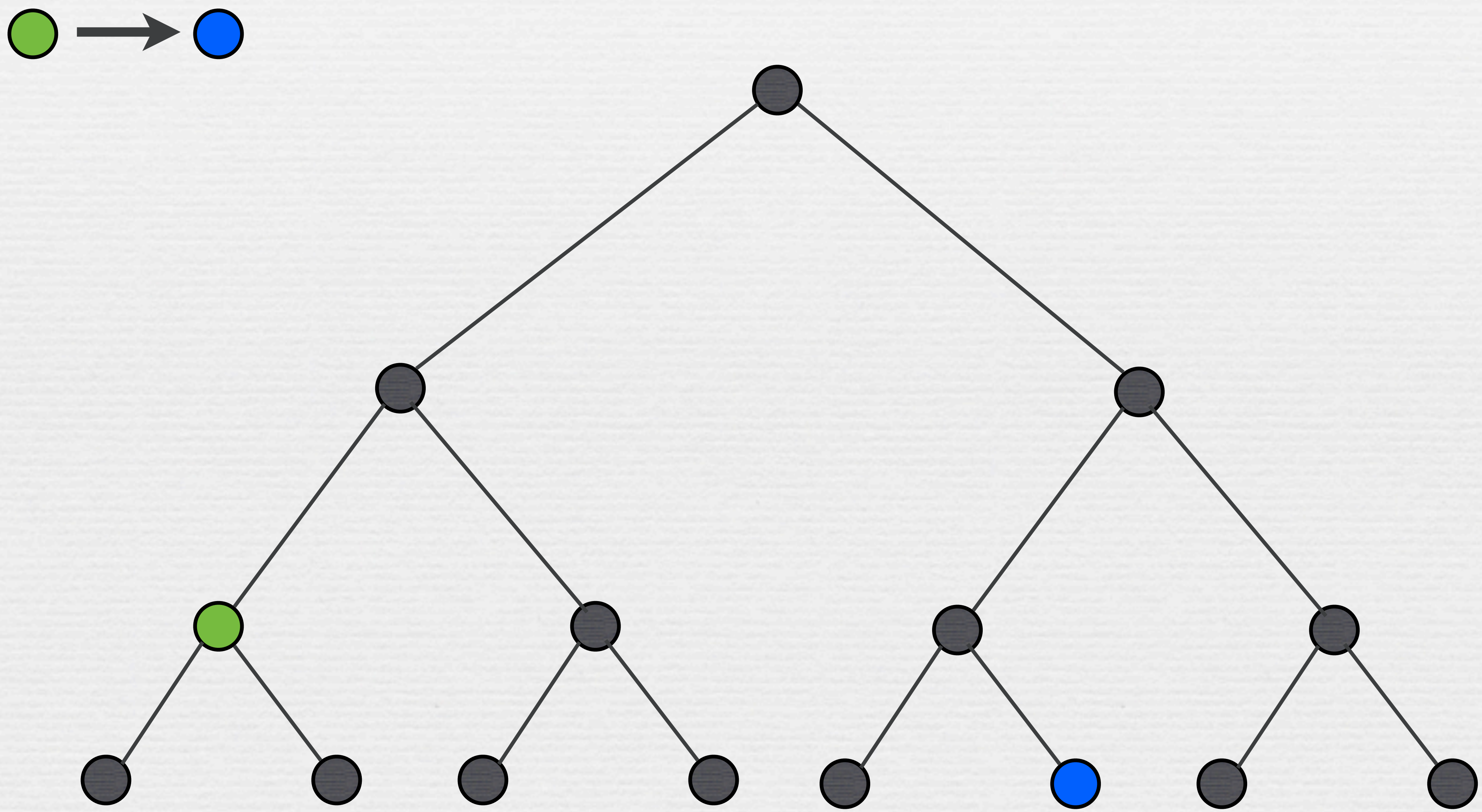
Construction on a Tree

- When G is a tree there exists an S s.t. (G,S) is shattered and internally connected and the membership dimension of S is O(diam(G)*log(n))
  - Is the log(n) factor on our tree construction necessary?

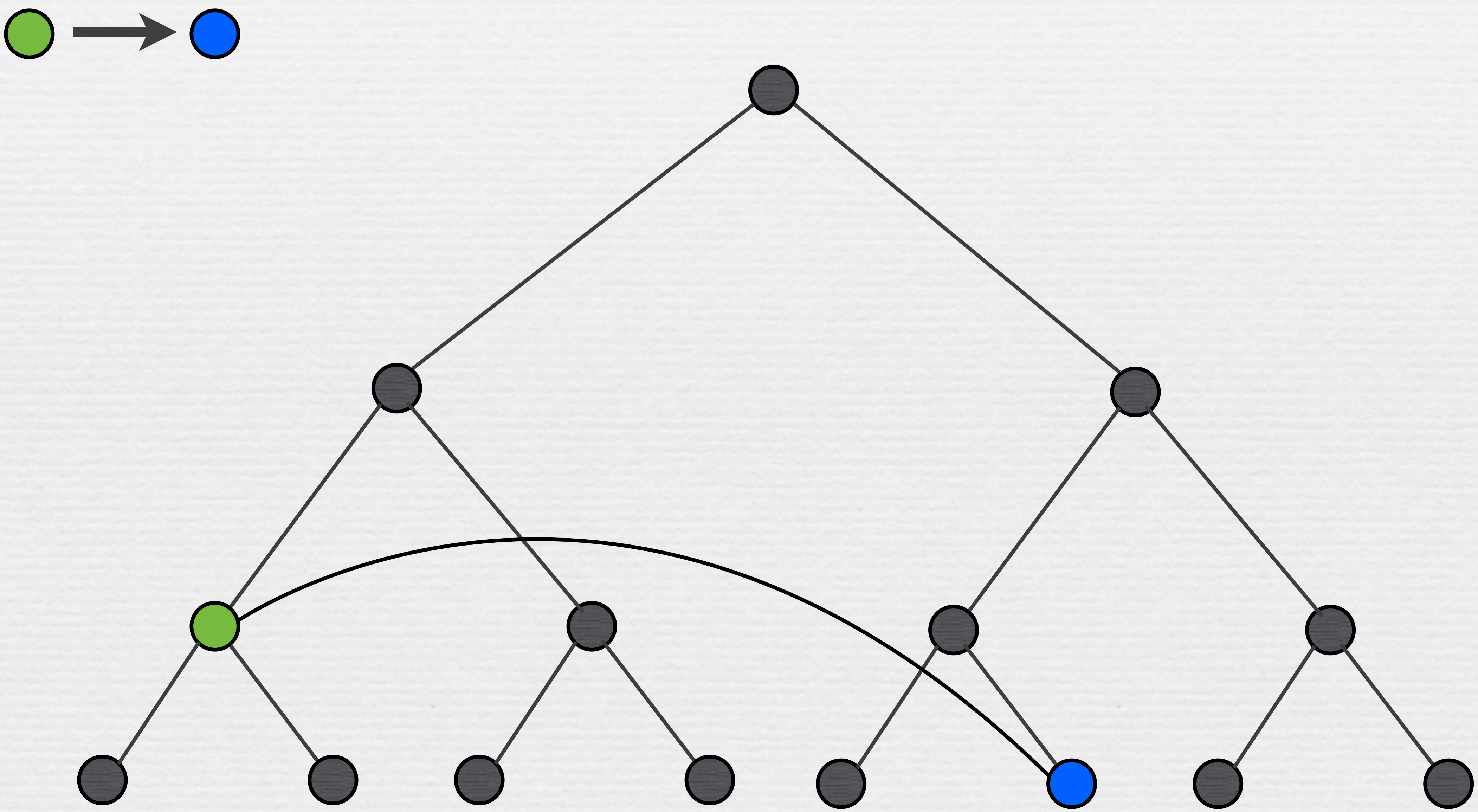
On Graphs

- To construct S for a general graph
  - Find a low diameter spanning tree
  - Use the described groupings to tree construction
- Can we do better on a general graph routing directly in G?

On Graphs



On Graphs



Discussion

- Open Questions
  - Is Internally Connected a necessary condition?
  - Is the log(n) factor on our tree construction necessary?
  - Can we do better on a general graph routing directly in G?

Thank you!