Imputing Missing Data in Sensor Networks via Markov Random Fields

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Scalable Methods for the Analysis of Network-Based Data

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Main Ideas

- Efficiently impute missing data in network
  - Markov blankets
  - Conditionally independent subsets
- Allow unknown model parameters

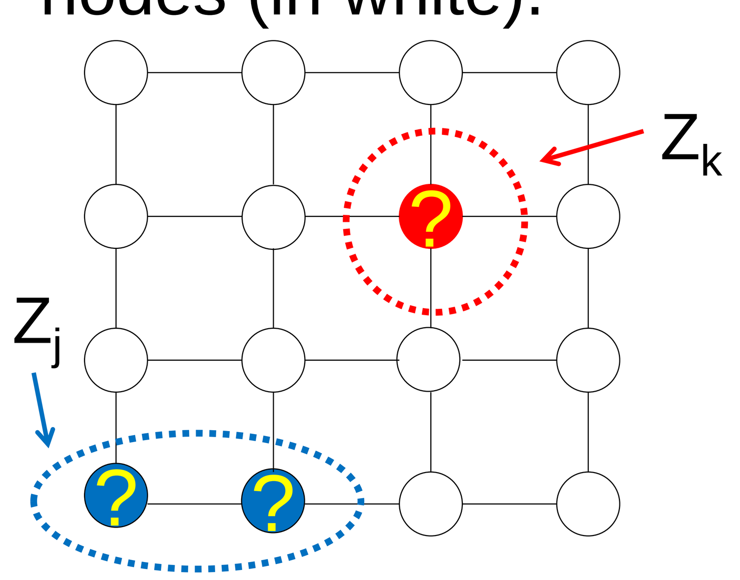
Conditional Independence in Markov Random Fields (MRFs)

Basic MRF:

- Nodes correspond to measurements
- Edges connect directly related nodes

Conditional Independence (C.I.):

- Denote set of unobserved nodes as Z
- Denote set of observed nodes as X (white)
- Missing sets Z_j and Z_k are C.I. if all paths between the two are "cut off" by observed nodes (in white).



Example MRF with two C.I. sets (Z_j and Z_k)

Markov Blankets

- Set of observed nodes "surrounding" missing set Z_j is the *Markov blanket*, denoted B_j.

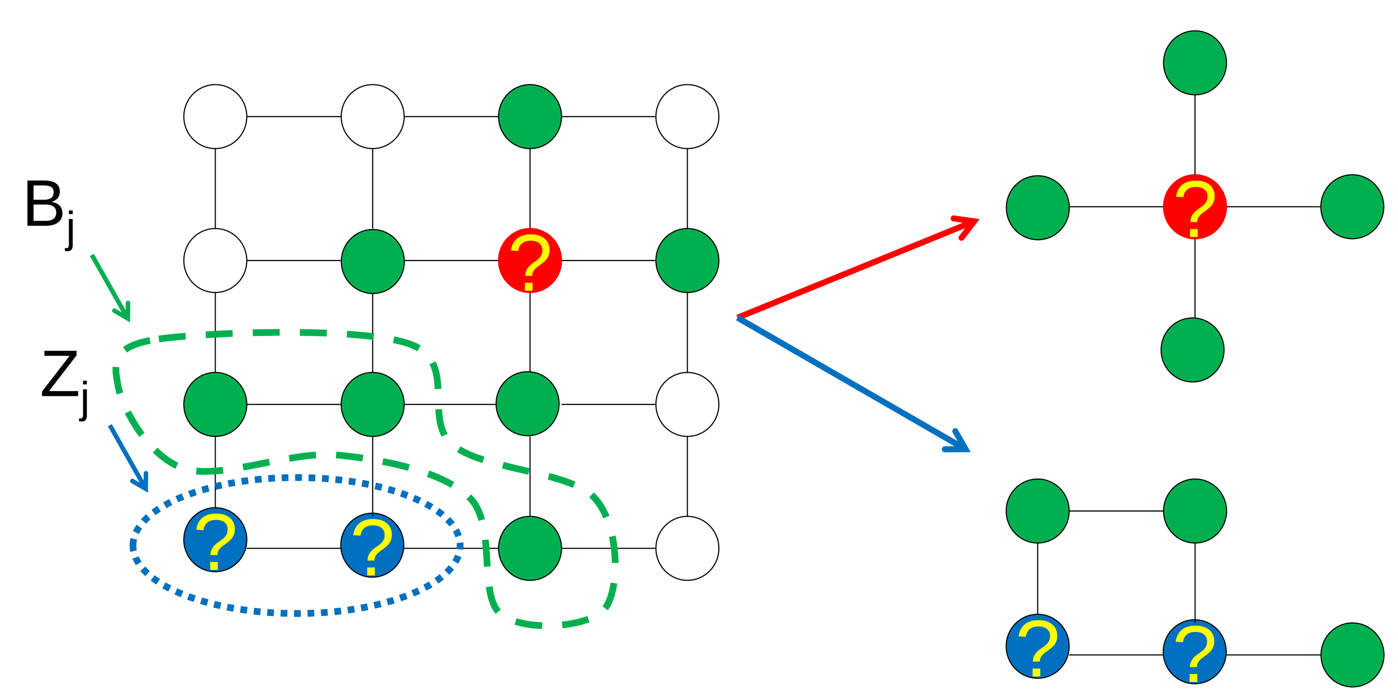
$$P(Z_j|X,\theta) = P(Z_j|B_j,\theta)$$

- Only conditioning on a set's Markov blanket can reduce computational complexity

Conditionally Independent Subsets

- Decomposition into conditionally independent sub-problems
- Can be combined with Markov blankets as shown below

left: *Markov Blanket of Z_j*
right: *Decomposition into C.I. sub-problems*



Computational Complexity

This gives three methods to calculate P(Z|X,ϴ). The bound on each method's computational complexity is below, along with the approximate number of operations required to find P(Z|X,ϴ) in the above graph.

1. Assume Gaussian Data

Method	Complexity	Small Example
Naive	$O(\|X\|^3)$	2197
MB	$O(\|B\|^3)$	343
MB/CI	$O(\max_i \|B_i\|^3)$	64

2. Assume Discrete Data (binary case, k=2)

Method	Complexity	Small Example
Naive	$O(k^{\|X\|})$	8192
MB	$O(k^{\|B\|})$	128
MB/CI	$O(\max_i k^{\|B_i\|})$	16

Parameter Estimation via EM

- Simultaneously estimate parameters and impute missing data using Expectation Maximization (EM) algorithm:

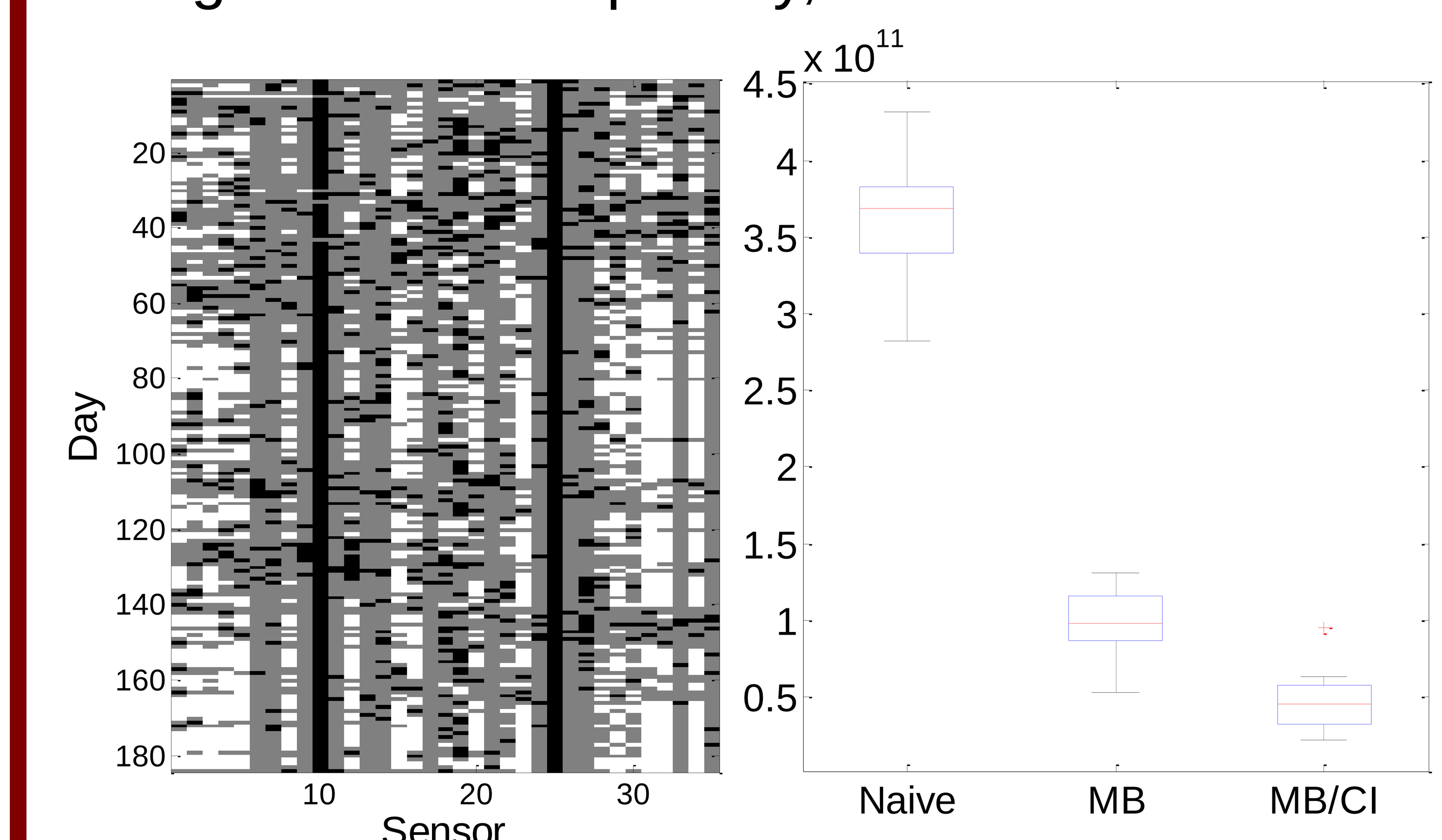
E Step Z^{t+1} = Most likely Z given X and Θ^t

M Step Θ^{t+1} = Most likely ϴ given X and Z^{t+1}

- E-step is equivalent to previously discussed methods for imputing missing data

Example: Rainfall Measurements

- 35 rainfall sensors across northern India
- MRF has a node for each node/day combination in a 122 day season.
- Edges are KNN spatially, Markov in time.



Left: *Missing data pattern (X=white,Z=black,B=gray)*
Right: *Complexity of imputation methods*

Current Work

- Efficiently balance multiple C.I. sub-problems during imputation
- Learning the edge structure of the MRF